Efficient Asset Allocations in the Banking Sector and Financial Regulation*

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Abstract

The failure of a bank or a bank experiencing a crisis usually has negative spillovers for other banks in the economy, such as through informational contagion or an increased cost of borrowing. Such spillovers are likely be higher when the other banks are close to failure as well. This paper shows that this gives rise to externalities among banks which arise from their portfolio choices. The reason is that the assets a bank holds on its balance sheet determine in which situations a bank is going to be in a crisis, and thus also whether other banks are then in a crisis as well. As a result, the equilibrium portfolio allocations in the economy are typically not efficient. Some banks may choose too correlated portfolios, but others also too heterogenous portfolios. The optimal regulatory treatment of banks is typically heterogenous and may involve encouraging more correlation at already highly correlated banks but lowering correlation at other banks. Additional inefficiencies arise when bank failures have also implications outside the banking sector. Overall, the paper highlights a role for regulation in a financial system in which the costs of financial stress at institutions are interdependent.

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1 Introduction

The failure of a bank often induces negative spillover effects for other banks. Such spillovers may, for example, be of informational nature. Depositors, having observed the failure of one bank, may conclude that other banks are also in troubles and may start running on them as well. The other banks may also suffer from higher borrowing costs because lenders generally update their beliefs about the riskiness of banks. There may also be direct costs arising from the failure of a bank, for example from defaulting interbank loans. Another channel of spillovers is through asset prices. The liquidation of assets following the failure of a bank may depress asset prices, and hurt other banks that have the same assets on their balance sheets.

The overall impact of such spill-overs is likely to depend on the general health of the other banks. A bank that is already on the brink to failure may particularly suffer from such spillovers and may even fail itself as a consequence. And when many banks to fail at the same time (systemic crisis), there may additionally be costs for society as whole, for example because it then becomes more difficult for firms to obtain financing. By contrast, the impact of such spillovers at healthy banks is likely to be limited. Healthy banks may even benefit from crisis at their competitors, either because it enables them to capture a higher market share or by being able to purchase up their assets at discounted prices.

The main idea of this paper is as follows. When the externalities from a bank's failure depend on whether other banks are in troubles at the same time, then a bank's portfolio choice will have welfare implications beyond the traditional risk-liquidity trade-off. The reason is that the assets a bank holds on its balance sheet determine in which situations the bank is going to be in a crisis, and thus also whether this will be at times other banks are in troubles as well. For example, if a bank invests in the same portfolio as most of the other banks, it would tend to fail at a time other banks are failing as well, thus potentially maximizing the negative spillovers. By contrast, a bank that holds a portfolio very different from other banks, even if it is very risky itself, may overall pose very little externalities since it will tend to fail at a time other banks are in good shape (and potentially even can purchase its assets).

We consider a setup where there are interbank externalities due to liquidation costs. They arise because a bank's failure depresses asset prices, which negatively affects all other banks that are in troubles at the same time, since these banks also have to liquidate. There are many banks in our economy, each of which decides on which combination of risky assets it wants to hold in its portfolio. We show that the equilibrium in this economy is typically not efficient due to the presence of externalities. Suppose a bank invests more in an asset. It then makes itself more similar to all banks that are relatively exposed to this asset. This is costly for the latter since it now becomes more likely that they have to liquidate jointly with the bank. But there is also an counteracting externality. When a bank invests in the asset, it also makes itself less similar to all banks which are less exposed to this asset. This benefits these banks by reducing their likelihood of joint liquidation.

We show that the net effect of these externalities is ambiguous. As a result of this, a bank may either be too close to the average portfolio in the banking system (too correlated) but also be too far away (too heterogeneous). We also show that typically some banks are always not enough correlated with the average portfolio, while at the same time others are too much correlated. Optimal financial regulation should thus treat banks heterogeneously. In particular, we show that, perhaps paradoxically, correlation at already correlated banks may be further encouraged, while correlation at less correlated banks should be discouraged.

The reason for this result is as follows. Consider a bank which is relatively specialized (that is, not very correlated with the other banks in the banking system). If this bank gets closer to the average portfolio in the economy it will become more similar to most other banks in the system, which will impose an overall large externality. It is true that it will also increases differences with the banks which are even more specialized into the asset this bank is invested in. However, these are only a few banks. Thus, the negative externality is relatively large compared to the positive one, and hence this bank may correlate more than what is optimal. For a bank which is already fairly correlated, the opposite reasoning can be applied.

These considerations concern the impact a bank's portfolio choice has on other banks. However, the effects of a banks' failure are not only limited to the banking sector itself. They will also affect the returns of the agents who may purchase the assets in a liquidation (in our model these are located outside the banking sector). Furthermore, there may be effects in the wider economy. For example, when there are many failures at the same time ("systemic crisis"), this may induce social costs in the form of a credit crunch. We show that the overall efficiency implications of banks' portfolio choices depend crucially on how these effects materialize. In the limit case when there are no losses from transferring assets to outsiders and there are no social externalities, the equilibrium is always efficient. In the case where the transfer of an asset to outsiders incurs a deadweight loss which increases in the total amount of assets liquidated, we find that banks tend to correlate "too little". However, when there are social costs from a joint failure of banks, there is a tendency for "too much" correlation in the financial system.

Concluding, the analysis in this paper suggests that the consequences of banks' portfolio choices for the financial system are complex and go beyond the classical risk-liquidity trade-off. This is because in a financial system in which there are various interlinkages across banks, the social cost of risk at a bank is not only determined by its likelihood of failure, but crucially also by in what situations it is likely fail. In particular, banks which invest in idiosyncratic portfolios are relatively more likely to fail at times in which other banks are in good health and their failure is hence less likely to pose large externalities. By contrast, the externalities of banks whose investment is closer to their average portfolio in the economy may be larger. Financial regulation that operates from a systemic viewpoint should strive to take this into account.

The paper proceeds as follows. The next section discusses related literature. Section 3 sets up the model. Section 4 analyzes the efficient allocation of portfolios in the economy. In Section 5 the implications for the optimal regulatory treatment of bank portfolios are discussed. The final section concludes.

2 Related Literature

There is a rapidly growing literature which analyzes banks' portfolio choices. Several papers have in particular considered the implications for banks' incentives to hold liquidity in the presence of fire-sales of assets.[1] Gorton and Huang (2004) present a model in which financing is restricted by agency problems at the level of the firm, as in Holmström and Tirole (1998). Banks supply liquidity in order to be able to buy up assets cheaply in a crisis. This is socially inefficient because a private provision of liquidity implies less investment in risky, high return, assets. In Wagner (2007) banks also hold liquidity in order to profit from fire-sales. Purchasing banks, however, are inferior users of the assets compared to the originating bank. Because of this they do not perceive the full social value of providing liquidity and, as a result, liquidity is underprovided in equilibrium. Acharya, Shin, Yorulmazer (2007) show that liquidity can be either under- or overprovided in the presence of fire-sales. The reason is that risky assets -the alternative to holding liquidity- can be used as well to purchase assets in a crisis because they serve as a collateral. The relative incentives to hold liquidity, as opposed to investing in the risky asset, depend then on the pledgeability of assets. If the latter is high, liquidity is underprovided in equilibrium.[2]

This paper differs from these papers in that it analyzes banks' choices between two risky assets, rather than between a risky and a safe asset. Such a choice is also considered in various papers by Acharya and Yorulmazer. In their papers banks have incentives to choose overly correlated assets. In Acharya (2001) and Acharya and Yorulmazer (2005), this is because bank owners invest in correlated assets because they do not internalize the costs of a joint failure due to limited liability. In Acharya and Yorulmazer (2006) and Acharya and

[1] Allen and Gale (2004), by contrast, consider the impact on the incentives of outsiders to provide liquidity.

[2] Perotti and Suarez (2002) present a model where banks also gain from the failure of their competitors. They do this not through fire-sales, but by capturing a higher market share. This is shown to make banks more prudent in their lending behavior (the analog to holding more liquidity in the former papers).

Yorulmazer (2007) banks want to increase the likelihood of failing simultaneously in order to induce a regulator to bail them out, which again causes them to correlate too much.

A difference to these papers is that in our setup banks dislike being correlated with each other due to the higher costs of failure this implies. Still, a failure that takes place when many other banks fail as well imposes greater externalities than if only a few other banks are failing. Interestingly, however, and in contrast to the above papers this does not imply that banks choose too correlated portfolios. The reason is that in our setting banks can hold a mix of assets (while in the papers by Acharya and Yorulmazer the investment decision is an "either-or" choice). Consider a bank which is invested in two assets X and Y, but more so in asset X. If the bank chooses a more correlated portfolio by investing more in Y (correlated in the sense of choosing a portfolio which is closer to the average portfolio in the banking sector, which is the portfolio that combines X and Y equally in the case that portfolio allocations in the banking sector are symmetric), the bank becomes more similar to all banks invested more heavily in Y. However, it also becomes less similar to all banks which are even less invested in Y. As we show in the paper, this makes the overall externalities that arise from moving closer to the average portfolio ambiguous. As a result, banks may either correlate too much, but also too little in equilibrium.

While in the present paper, and in the papers by Acharya and Yorulmazer, a higher correlation among banks is undesirable, Wagner (2008a) presents a model where this is not case. The reason is that when banks are more correlated, their liquidity positions in a crisis will be more homogenous. There is then less need to reallocate liquidity through the interbank market, which improves welfare since the interbank market may not work perfectly in times of crisis. Moreover, a higher homogeneity of banks also reduces the need to regulate banks. This is because when banks are more homogenous they can rely less on interbank risk sharing. This in turn reduces any externalities that arise from such risk-sharing. A downside of the homogenization, however, is that it also lowers banks' incentives to hold liquidity and increases their incentives to invest in risky assets (while the total amount invested in risky asset stays constant in the present paper).

3 The Model

The economy is inhabited by a continuum of banks of mass 1 which are indexed by i. Each bank has collected one unit of funds from investors, of which a share d is in the form of deposits and $1-d$ is equity. Shareholders and depositors are both risk-neutral.

There are three dates. At date 1 banks can divide their funds between two assets, X and Y. We denote with $\alpha_i \in [0,1]$ the share invested by bank i in asset Y. $1-\alpha_i$ is then the share invested in $\dot{X}$. The assets mature at date 3. We denote with x and y the date-3 returns on asset X and Y, respectively. These returns are identically and independently distributed on $[0,\infty)$ according to a density $\phi(.)$.

At date 2, the date-3 returns x and y become known. The fundamental value of bank i is then given by

$$v_i = (1-\alpha_i)x + \alpha_i y. \tag{1}$$

Following this, a bank run occurs at a bank if the (fundamental) value of its assets falls below the value of deposits. That is, bank i experiences a run iff $v_i < d$ (we thus rule out panic runs). The bank then has to sell its entire portfolio to investors, which are located outside the banking system.[3] We assume that the portfolio can only be sold at a discount $C(.) \geq 0$ to its date-3 value. $C(.)$ is strictly and smoothly increasing in the number of portfolios which are liquidated at the same time. We also assume that $C(0) = 0$, that is, if the mass of other selling banks is zero, the portfolio can be sold without a loss.

The increasing discount may stem from several sources. First, there may be fire-sale prices due to *cash-in-the-market pricing* (e.g., Allen and Gale, 2004, Schnabel and Shin, 2004, and Gorton and Huang, 2004): when the total supply of liquidity by outsiders is limited, all available liquidity may have to be used to purchase assets. This then necessarily implies that if more portfolios are sold, the price per portfolio has to decline, which naturally gives rise to the above cost function. Alternatively, the purchasers of the portfolios may for

[3] Banks themselves cannot purchase assets as they do not hold liquidity. The impact of fire-sales on their incentives for holding liquidity has, for example, been emphasized in Acharya, Shin and Yorulmazer (2007).

example require a higher compensation when they have to hold more assets (e.g., because they are risk-averse), also implying that the price per portfolio is declining in the amount of portfolios sold.

The crucial assumption is here that C, which represents a bank's cost of failure, is higher when other banks face stress at the same time. Besides fire-sales, this may be for a variety of other reasons. For example, there may be informational spillovers from the failure of a bank, driving up the cost of borrowing at other banks. Such a spillover is likely to hurt a bank more when it is close to failure itself. Or, there may be network externalities (such as from the failure of the settlement system) which will also tend to hurt banks more when they are weak.

Finally, at date 3 assets mature and shareholders and depositors consume their respective returns. Since both shareholders and depositors are risk-neutral, each bank's value is maximized when its overall expected return is maximized. The latter can be split into the expected fundamental value of a bank's portfolio (v_i) minus the expected liquidation costs due to the asset discount. Since both assets are identically distributed, a bank's portfolio allocation α_i does not influence its expected fundamental value v_i. Hence, a bank's value is simply maximized when its expected liquidation costs are minimized (which we derive below).

4 Efficient Portfolio Allocations in the Banking Sector

An allocation in this economy can be summarized by a density function $f(\alpha)$ on $[0, 1]$, which gives the density of all banks playing the portfolio α. Alternatively, the allocation can be represented by the corresponding mass function $F(\alpha)$, which represents the mass of banks playing α or less. In this section we analyze the $F(\alpha)$ which is efficient for the banking sector, that is the $F(\alpha)$ which minimizes the total expected liquidation costs in the banking sector. We denote this mass function with $F^e(\alpha)$.[4]

We first derive a single bank's expected liquidation costs from playing α. Recall that a bank has to sell its portfolio if the fundamental value of its assets is less than the deposits. From rearranging (1) we have that the bank has to liquidate iff $y < \widehat{y}(x)$, where $\widehat{y}(x)$ is given by

$$\widehat{y}(x, \alpha) = \frac{d}{\alpha} - \frac{1-\alpha}{\alpha}x. \tag{2}$$

$\widehat{y}(x)$ gives us the critical return of asset Y for which the bank just survives if asset X pays x.

Suppose now that we have $y < \widehat{y}(x)$, that is the bank has to liquidate. What are the costs the bank will incur? This will depend on how many other banks are failing at the same time. Consider first $x > y$, that is asset X has a higher return than asset Y. In such a situation, banks with low α (that is, banks which have not invested much in Y) will survive, but banks with high α will fail. It follows there is a critical value $\widehat{\alpha} = \widehat{\alpha}(x, y)$, such that all banks with $\alpha > \widehat{\alpha}(x, y)$ fail, while all banks with $\alpha \leq \widehat{\alpha}$ survive. From (1) we have that $\widehat{\alpha}$ is implicitly defined by $v = (1 - \widehat{\alpha})x + \widehat{\alpha}y = d$. Rearranging for $\widehat{\alpha}$ we get

$$\widehat{\alpha}(x, y) = \frac{d - x}{y - x}. \tag{3}$$

Since all banks with $\alpha > \widehat{\alpha}(x, y)$ fail, the mass of failing banks is $1 - F(\widehat{\alpha}(x, y))$ and the liquidation costs are consequently $C(1 - F(\widehat{\alpha}(x, y)))$. Consider next $x < y$. Now all banks which are more exposed to X will fail, that is banks with $\alpha < \widehat{\alpha}$. The liquidation costs in this case are hence $C(F(\widehat{\alpha}(x, y)))$.

The liquidation costs that arise for a return realization (x, y) can thus be summarized as follows:

- if $y \geq \widehat{y}(x)$: no liquidation costs,
- if $y < \widehat{y}(x)$ and $x > y$: liquidation costs are $C(1 - F(\widehat{\alpha}(x, y)))$,

[4] We only characterize the efficient aggregate allocation $F^e(\alpha)$, which completely suffices for our purpose. A bank's individual allocation α_i at the efficient solution can in fact not be uniquely determined, as will become clear soon.

- if $y < \widehat{y}(x)$ and $x < y$: liquidation costs are $C(F(\widehat{\alpha}(x,y)))$.

A bank's total expected liquidation costs can then be found by integrating over the liquidation costs for each return realization (x, y) weighted by its density $\phi(x)\phi(y)$. The total expected liquidation costs in the banking sector are then obtained by integrating over the expected liquidation costs of all banks.

Appendix A makes assumptions on the distribution function ϕ. These assumptions ensure that *i)* more investment in one asset increases the likelihood of failure if this asset performs worse than the other one but reduces the likelihood of failure when it performs better, *ii)* moving the portfolio towards $\alpha = 1/2$ always reduces the likelihood of failure, *iii)* the marginal gains from doing so (in terms of reducing the likelihood of failure) are declining the closer we get to $\alpha = 1/2$.

Proposition 1 *The efficient portfolio allocation $F^e(\alpha)$ is strictly increasing in α.*

Proof. *See Appendix B.* ■

Since the mass function is strictly increasing, it follows that the efficient allocation requires that all portfolio allocations are played by at least some banks. In particular, we also have banks that play $\alpha = 0$ and $\alpha = 1$, that is some banks hold completely polarized portfolios.

What is the intuition behind this result? Since the liquidation costs are increasing in the number of banks failing, it is not optimal to have many banks playing the same allocation. The reason is that banks would then tend to liquidate together, which would incur substantial liquidation costs. Since one wants to avoid pooling many banks in a liquidation, it is efficient to spread banks as widely as possible on the continuum $[0, 1]$.

5 Implications for Financial Regulation

Proposition 1 has shown that it is optimal for the banking sector as a whole that all possible portfolios are played by the banks. There is no guarantee, however, that banks themselves will choose the optimal allocation. The equilibrium may in fact be inefficient, and thus gives rise to a role for financial regulation. Let us first consider efficiency within the banking system and ignore the impact banking failures and fire-sales may have in other parts of the economy. This *interbank* aspect of efficiency can be analyzed by studying whether there are any externalities among banks. Clearly, in the absence of such externalities the equilibrium would be efficient within the banking sector.

To this end, consider the impact of a bank i bank becoming more correlated on the other banks in the economy. For this we presume that the aggregate portfolio holdings in the economy are symmetric (that is we have $f(\alpha) = f(1-\alpha)$). The average portfolio in the banking sector consists then of equal parts of X and Y. Getting more correlated for a bank thus implies moving closer towards $\alpha = 1/2$. Suppose, in particular, that bank i slightly increases its α_i starting from an α_i of less than 1/2. That is, the bank gets more correlated by investing more in asset Y. When it does so, it tends to fail more often when $x > y$, and less often $x < y$. In the situations where the bank now fails (although it did not so previously), the (per portfolio) liquidation costs in the banking sector rise marginally. This is because C is increasing in the mass of failing banks. This poses a negative externality an all banks that fail at the same time. These are all the banks that are even more exposed to Y, that is all banks that play an α higher than α_i. The total externality from this on the other banks' liquidation costs can be shown to be (second part of equation 16 in Appendix B):

$$C'(1 - F(\alpha_i))B'(\alpha_i)(1 - F(\alpha_i)) > 0 \tag{4}$$

The externality is thus the product of the impact of more liquidations on the liquidations costs, $C'(1 - F(\alpha_i))$, the probability of the area where more liquidations take place $B'(\alpha_i)$ ($B'(\alpha_i) > 0$, see Appendix A), and the mass of banks which play an α higher than α_i, $1 - F(\alpha_i)$.

Conversely, liquidation costs are lower in the situations where the bank now survives. This induces a positive externality on all banks with $\alpha < \alpha_i$ since such situations occur when $x < y$. The total externality from this is (first part of equation 16 in Appendix B):

$$C'(F(\alpha_i))A'(\alpha_i)F(\alpha_i) < 0 \tag{5}$$

This externality now depends on the mass of banks which play a higher α and is positive because it reduce the expected liquidation costs ($A'(\alpha) < 0$, see Appendix A).

It can be easily seen that the net effect from these externalities cannot be generally determined. This is because their relative magnitude crucially depends on how many of the other banks play a higher and how many play a lower α (which in turn depends on the α_i played by bank i and the $\alpha's$ of all other banks). Suppose for example that all banks play a higher α than bank i. We then have $F(\alpha_i) = 0$. Then, there is obviously no positive externality, and the negative externality prevails. In such a situation bank i is too much correlated from an efficiency perspective.[5] However, when many banks play a lower α, $F(\alpha_i)$ is large. Additionally we have that $|A'(\alpha_i)| > B'(\alpha_i)$ because moving more towards the fully diversified portfolio reduces the overall probability of the bank being liquidated (see Assumption (ii) in Appendix A). The positive externality may then prevail. In such situations, regulators should optimally encourage correlation at this bank. This suggests that regulators should, perhaps paradoxically, discourage correlation at uncorrelated banks (low α_i relative to other banks). At more correlated banks by contrast, also the positive externality is present. It may hence be optimal to further encourage their correlation.

Note that in our setup being more similar and being more diversified is the same thing. This is because when banks are symmetrically distributed around $\alpha = 1/2$ ($f(\alpha) = f(1-\alpha)$), the consolidated portfolio in the banking sector is the fully diversified portfolio, no matter whether banks are individually diversified or not. We can thus rephrase the last sentence as saying that diversification should be encouraged at relatively diversified banks (thus making them even more diversified), while it may be optimal to discourage diversification at banks which are already close to the fully polarized portfolio.

Let us next consider overall efficiency, thus also taking into account the effect of bank correlation on other parts in the economy, and in particular also on the purchasers of the

[5] A similar result is obtained in the various papers by Acharya and Yorulmazer. They show (in a two-bank setup) that investing in the same asset as the (single) other bank increases the likelihood of joint failure of the two banks, which in turn may impose negative externalities on society. Thus, banks correlate (“herd” in their papers) too much.

assets. As we will see, the results will crucially depend on how these effects materialize.

Suppose first that the outsiders who purchase the assets are risk-neutral as well and that their (exogenous) supply of liquidity leads to cash-in-the-market pricing, CITMP. CITMP refers to situations where the total amount of liquidity provided by outsiders is less then the combined fundamental value of all portfolios that have to be liquidated. Assume that the transfer of assets to the outsiders does not result in any efficiency losses, that is outsiders can extract their full fundamental value. Moreover, assume that there are no other deadweight losses from banking failures, which may realize either at banks themselves (e.g., due to bankruptcy costs) or in other parts of the economy (e.g., because of a credit crunch).

Under CITMP, banks will make a loss from the liquidation of their assets. However, this loss is then exactly offset by the gains to the outsiders since there is no overall efficiency loss from transferring assets: fire-sales are a zero-sum game. It follows that, conditional on the liquidity supplied by outsiders, all portfolio allocations are efficient. Thus, there is no rationale for regulation in this case.

Suppose next that there is an efficiency loss from selling assets to outsiders in the form of a constant loss $\delta > 0$ per portfolio transferred. This loss may occur because outsiders, who do not have the specialized knowledge of banks, are less efficient users of the asset. In this case the total efficiency losses in the economy are proportional to the number of portfolios that are liquidated. Welfare hence decreases in the number of banks that have to liquidate. Since higher correlation minimizes the likelihood of liquidation at each bank (since it implies a more diversified portfolio), it follows that full correlation (or a complete homogeneity of banks) is the socially efficient outcome. Banks itself, however, will not find it optimal to play all the full correlated outcome. The reason for this is the same as for Proposition 1, which showed that a heterogeneity of portfolios is optimal for the banking sector. If banks were indeed all holding the same portfolio, they would need to liquidate at the same time, which would be very costly for them.[6] In this case, too little correlation

[6]Note that the assumption that a bank's liquidation costs are increasing in the number of banks failing at the same time (which is required for Proposition 1) is still fulfilled in this context since δ is a constant

is the likely outcome for the banking sector.

Finally, consider a different scenario. Assume that there are no efficiency losses from transferring assets ($\delta = 0$ in the notation of the previous paragraph). However, there are social costs of a systemic crisis: when more than a certain number of banks fail at the same time, there are costs $C_S > 0$ outside the banking sector (this may be, for example, because there is then a credit crunch, which causes losses to the wider economy). More specifically, assume that a systemic crisis arises when the mass of failing banks is larger than F_S ($F_S < 1/2$). Also assume that the social costs increase in the mass of failing banks ($C'_S(.) > 0$) and that the marginal social costs are constant $C'_S(.) = \overline{C}'_S$.

A bank that plays α_i then always fails in a systemic crisis (and thus always amplifies the systemic costs when it fails) when both $F^e(\alpha_i) \geq F_S$ and $1 - F^e(\alpha_i) \geq F_S$. This is because if the bank fails when $x < y$ it fails together with $F^e(\alpha_i)$ banks, while if it fails when $x > y$, it fails together with $1 - F^e(\alpha_i)$ banks. Defining with $F^{e-1}(.)$ the inverse of $F^e(.)$, this happens when the bank's α_i fulfills $F^{e-1}(F_S) \leq \alpha_i \leq F^{e-1}(1 - F_S)$, that is when the bank is relatively correlated. For α_i outside this range, the bank does not contribute to a systemic crisis if either $x > y$ or $x < y$.

Since the systemic costs occur outside the banking system, they are not internalized by banks. They hence do not affect the equilibrium amount of correlation in the banking sector. Suppose that, starting from an equilibrium, we reduce correlation at a relatively correlated bank. In particular, we lower correlation at a bank with $\alpha_i = F^{E-1}(F_S)$ to $\alpha_i = 0$. In the situations of additional failures (which occur when $x < y$), the bank then fails without a systemic crisis since the number of other banks failing at the same time is less than F_S. In the situations of avoided failures (which arise when $x > y$), the bank previously failed in a systemic crisis. Hence, the bank overall contributes less to systemic crises. Therefore, the negative externality (in terms of amplifying systemic crises) posed by the bank declines and efficiency increases. Thus, in this scenario banks may be too similar in equilibrium. Encouraging heterogeneity in the banking system may then improve welfare.

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and CITMP itself implies increasing costs.

6 Concluding Remarks

This paper has considered the efficiency of banks' portfolio choices when the externalities from a bank's failure depend on the general health of the banking sector. We have shown that there are interesting implications for financial regulation. For example, encouraging less correlation at banks is not necessarily desirable for the financial system. There are plausible scenarios under which banks may either chose too much or too little correlation in equilibrium.

Our analysis has also shown that the welfare implications of a change in an individual bank's portfolio depend crucially on how the bank's portfolio relates to the portfolios of the other banks in the financial system. This is because when a bank invests more in an asset, this has a negative effect on all banks which are even more invested in this asset but a positive effect on all banks which are less invested in the asset. The net-effect then depends on the relative size of these two groups of banks and thus on the portfolio allocation of the bank itself. Since banks will typically hold different portfolios in equilibrium, their regulatory treatment should hence not be the same. Indeed, as we have shown, it may be optimal to encourage correlation at some banks, while discouraging it at others.

References

[1] Acharya, V.: 2001, 'A Theory of Systemic Risk and Design of Prudential Bank Regulation'. *mimeo, London Business School.*

[2] Acharya, V., H. Shin, and T. Yorulmazer: 2007, 'Endogenous Choice of Bank Liquidity: The Role of Fire Sales'. *working paper, Princeton University.*

[3] Acharya, V. and T. Yorulmazer: 2005, 'Limited Liability and Bank Herding'. *mimeo, London Business School.*

[4] Acharya, V. and T. Yorulmazer: 2006, 'Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures'. *forthcoming, Review of Financial Studies.*

[5] Acharya, V. and T. Yorulmazer: 2007, 'Too Many to Fail - An Analysis of Time-Inconsistency in Bank Closure Policies'. *Journal of Financial Intermediation* **16**, 1–31.

[6] Allen, F. and D. Gale: 2004, 'Financial Fragility, Liquidity, and Asset Prices'. *Journal of the European Economic Association* **2**, 1015–1048.

[7] Gorton, G. and L. Huang: 2004, 'Liquidity, Efficiency, and Bank Bailouts'. *American Economic Review* **94**, 455–483.

[8] Holmström, B. and J. Tirole: 1998, 'Private and Public Supply of Liquidity'. *Journal of Political Economy* **106**, 1–40.

[9] Perotti, E. and J. Suarez: 2002, 'Last Bank Standing: What Do I Gain If You Fail?'. *European Economic Review* **46**, 1599–1622.

[10] Schnabel, I. and H. S. Shin: 2004, 'Liquidity and Contagion: The Crisis of 1763'. *Journal of the European Economic Association* **2**, 929–968.

[11] Wagner, W.: 2007, 'Aggregate Liquidity Shortages, Idiosyncratic Liquidity Smoothing and Banking Regulation'. *Journal of Financial Stability* **3**, 18–32.

[12] Wagner, W.: 2008a, 'The Homogenization of the Financial System and Liquidity Crisis'. *Journal of Financial Intermediation* **17**, 330–356.

Appendix A: The Assumptions on the Distribution Function ϕ

It is useful to define with

$$A(\alpha) := \int_0^d \left(\int_x^{\widehat{y}(x,\alpha)} \phi(x)\phi(y)dy \right) dx \tag{6}$$

the probability that a bank that plays α fails when there is $x < y$, and with

$$B(\alpha) := \int_0^d \left(\int_0^x \phi(x)\phi(y)dy \right) dx + \int_d^{\widehat{x}(0,\alpha)} \left(\int_0^{\widehat{y}(x,\alpha)} \phi(x)\phi(y)dy \right) dx \tag{7}$$

the probability that the bank fails when $x > y$. $\widehat{x}(0)$ is the x at which $\widehat{y}(x)$ becomes zero. $\widehat{x}(0)$ is obtained by setting $\widehat{y}(x) = 0$ and solving for x:

$$\widehat{x}(0, \alpha) = \frac{d}{1 - \alpha}. \tag{8}$$

$A(\alpha) + B(\alpha)$ then gives us the overall probability of failure when α is played.

The assumptions on ϕ are:

(i) The density is smooth and has full support on $[0, \infty)$, that is we have $\phi(z) > 0$ for $z \in [0, \infty)$. From this follows that $A'(\alpha) < 0$ and $B'(\alpha) > 0$, that is more investment in Y makes it less likely that the bank fails if $x < y$ and more likely if $x > y$.

(ii) We have $A'(\alpha) + B'(\alpha) < 0$ for $\alpha < 1/2$ and $A'(\alpha) + B'(\alpha) > 0$ for $\alpha > 1/2$. This guarantees that more diversification (that is, moving towards $\alpha = 1/2$ from either side) always reduces the overall probability of failure.

(iii) We have $A''(\alpha) > 0$ and $B''(\alpha) > 0$. This ensures that the marginal impact of diversification $(A'(\alpha) + B'(\alpha))$ is declining in the amount of diversification.

Appendix B: Proof of Proposition 1

We first derive the total expected liquidation costs in the banking sector. Integrating over all liquidation outcomes we get that the liquidation costs for a bank which plays α are

$$\begin{aligned} K(\alpha) = \quad & \int_0^d \left(\int_x^{\widehat{y}(x,\alpha)} (\phi(x)\phi(y)C(F(\widehat{\alpha}(x,y))))\, dy \right) dx \\ + \quad & \int_0^d \left(\int_0^x (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \\ & + \int_d^{\widehat{x}(0,\alpha)} \left(\int_0^{\widehat{y}(x,\alpha)} (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx. \end{aligned} \tag{9}$$

The first integral in (9) refers to liquidations when $x < y$, while the second and the third integral refer to liquidations that arise when $x > y$. $\widehat{x}(0,\alpha)$ is the x at which $\widehat{y}(x,\alpha)$ becomes zero and is given by equation (8) in Appendix A. The total expected liquidation costs in the banking sector are then obtained by integrating over all banks:

$$\begin{aligned} TK &= \int_0^1 K(\alpha) f(\alpha) d\alpha \\ &= \int_0^1 \begin{pmatrix} \int_0^d \left(\int_x^{\widehat{y}(x,\alpha)} (\phi(x)\phi(y)C(F(\widehat{\alpha}(x,y))))\, dy \right) dx \\ + \int_0^d \left(\int_0^x (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \\ + \int_d^{\widehat{x}(0,\alpha)} \left(\int_0^{\widehat{y}(x,\alpha)} (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \end{pmatrix} f(\alpha) d\alpha \end{aligned} \tag{10}$$

The efficient $F^e(\alpha)$ is the $F(\alpha)$ which minimizes the above expression.

We show next that F^e is strictly increasing at each $\alpha \in [0, 1]$. For this we first show that there is positive density around $\alpha = 0$ and $\alpha = 1$. Without loss of generalization focus on $\alpha = 0$. Suppose to the contrary that there is an interval to the right of 0 without density. We can then extend this interval until we reach the first α with positive density. Denote this α with α_1. We hence have that $F(\alpha) = 0$ for $\alpha < \alpha_1$ but that at least one bank plays α_1.

Suppose now that this bank plays $\alpha = 0$ instead. This has a potential impact on the bank itself, but also on other banks in the economy. Consider first the impact on the bank itself. The difference in its expected losses between playing α_1 and 0 are given by

$$\begin{aligned}
&\int_0^d \left(\int_x^{\widehat{y}(x,\alpha_1)} (\phi(x)\phi(y)C(F(\widehat{\alpha}(x,y))))\, dy \right) dx \\
&+ \int_0^d \left(\int_0^x (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \\
&+ \int_d^{\widehat{x}(0,\alpha_1)} \left(\int_0^{\widehat{y}(x,\alpha_1)} (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \\
&- \int_0^d \left(\int_x^{\infty} (\phi(x)\phi(y)C(F(\widehat{\alpha}(x,y))))\, dy \right) dx \\
&- \int_0^d \left(\int_0^x (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx. \qquad (11)
\end{aligned}$$

This simplifies to

$$\begin{aligned}
&\int_d^{\widehat{x}(0,\alpha_1)} \left(\int_0^{\widehat{y}(x,\alpha_1)} (\phi(x)\phi(y)C(1 - F(\widehat{\alpha}(x,y))))\, dy \right) dx \qquad (12) \\
&- \int_0^d \left(\int_{\widehat{y}(x,\alpha_1)}^{\infty} (\phi(x)\phi(y)C(F(\widehat{\alpha}(x,y))))\, dy \right) dx.
\end{aligned}$$

Note that $\widehat{\alpha}(x,y)$ in the integrals only varies between 0 and α_1. Hence, we have $F(\widehat{\alpha}(x,y)) = 0$ in the integrals. It follows that (12) can be simplified to

$$\begin{aligned}
& C(1) \int_d^{\widehat{x}(0,\alpha_1)} \left(\int_0^{\widehat{y}(x,\alpha_1)} \phi(x)\phi(y) dy \right) dx \\
& -C(0) \int_0^d \left(\int_{\widehat{y}(x,\alpha_1)}^{\infty} \phi(x)\phi(y) dy \right) dx \\
= \;& C(1) \int_d^{\widehat{x}(0,\alpha_1)} \left(\int_0^{\widehat{y}(x,\alpha_1)} \phi(x)\phi(y) dy \right) dx > 0. \qquad (13)
\end{aligned}$$

Thus the change to $\alpha_1 = 0$ lowers the bank's expected liquidation costs.

Consider next the impact on other banks in the economy. Due to the move to $\alpha = 0$, the bank now fails for $x < y$ when $0 \leq \widehat{\alpha}(x,y) < \alpha_1$, while it previously survived for these outcomes. No other bank fails in these situations since they all play at least α_1. Hence, these additional failures do not have an impact on the other banks in the economy. The bank, however, now also survives when $x > y$ and $0 \leq \widehat{\alpha}(x,y) < \alpha_1$, while it previously failed for such outcomes. This reduces the liquidation discount $C(.)$, which may, if anything, have a positive impact on other banks. It follows that the overall impact on the other banks cannot be negative. Since we have shown that the move to $\alpha = 0$ increases the bank's payoff, we thus conclude that the original allocation did not maximize welfare.

Thus, we know that there has to be a positive mass around 0 and 1 for an efficient solution. We show next that there cannot be an interval without mass on $(0, 1)$. From this follows that there is positive mass everywhere (and F strictly increasing). Suppose, to the contrary, that there is such an interval. We can then extend this interval until we reach the first α (on each side) played by some banks. This is possible since we have shown that there is mass on both sides of the boundaries of the interval. Let us call the $\alpha's$ on the lower and upper end of this interval $\underline{\alpha}$ and $\overline{\alpha}$ (with $\underline{\alpha} < \overline{\alpha}$). We thus have that $f(\alpha) = 0$ for $\alpha \in (\underline{\alpha}, \overline{\alpha})$ and $f(\underline{\alpha}), f(\overline{\alpha}) > 0$. At $\underline{\alpha}$, the expected liquidation costs in the banking sector should be non-decreasing in α, since otherwise welfare could be improved by increasing α at a bank which plays $\underline{\alpha}$ a bit.

We derive next the impact of an increase in α at a bank on the expected liquidation costs in the banking sector. For this we consider first the impact on the bank itself. The derivative of a bank's expected liquidation costs wrt. to its own α is

$$\begin{aligned} K'(\alpha) = & \int_0^d \left(\phi(x)\phi(\widehat{y}(x))C(1 - F(\widehat{\alpha}(x, \widehat{y}(x)))) \frac{\partial \widehat{y}(x)}{\partial \alpha} \right) dx \\ + & \int_d^{\widehat{x}(0)} \left(\phi(x)\phi(\widehat{y}(x))C(F(\widehat{\alpha}(x, \widehat{y}(x)))) \frac{\partial \widehat{y}(x)}{\partial \alpha} \right) dx. \end{aligned} \tag{14}$$

Substituting in $A'(\alpha)$ and $B'(\alpha)$ (which can be obtained from equations (6) and (7) in Appendix A) and using that $\widehat{\alpha}(x, \widehat{y}(x)) = \alpha$, we get the following equation

$$C(F(\alpha))A'(\alpha) + C(1 - F(\alpha))B'(\alpha). \tag{15}$$

In a similar fashion, the impact on the other banks in the economy can be derived from (10) to be

$$C'(F(\alpha))F(\alpha)A'(\alpha) + C'(1 - F(\alpha))(1 - F(\alpha))B'(\alpha). \tag{16}$$

Thus the total impact of a change in α at the bank on the expected liquidation costs is

$$C(F(\alpha))A'(\alpha) + C(1 - F(\alpha))B'(\alpha) + C'(F(\alpha))F(\alpha)A'(\alpha) + C'(1 - F(\alpha))(1 - F(\alpha))B'(\alpha) \tag{17}$$

Using this condition we can now write down the condition that increasing α at a bank which plays $\underline{\alpha}$ does not lower the expected liquidation costs in the economy:

$$C(F(\underline{\alpha}))A'(\underline{\alpha})+C(1-F(\underline{\alpha}))B'(\underline{\alpha})+C'(F(\underline{\alpha}))F(\underline{\alpha})A'(\underline{\alpha})+C'(1-F(\underline{\alpha}))(1-F(\underline{\alpha}))B'(\underline{\alpha}) \geq 0. \tag{18}$$

Likewise, reducing α at a bank that plays $\overline{\alpha}$ should also not reduce the total liquidation costs. This condition writes

$$C(F(\overline{\alpha}))A'(\overline{\alpha})+C(1-F(\overline{\alpha}))B'(\overline{\alpha})+C'(F(\overline{\alpha}))F(\overline{\alpha})A'(\overline{\alpha})+C'(1-F(\overline{\alpha}))(1-F(\overline{\alpha}))B'(\overline{\alpha}) \leq 0 \tag{19}$$

Noting that $F(\underline{\alpha}) = F(\overline{\alpha})$ (since there is no density on $(\underline{\alpha}, \overline{\alpha})$), we can combine these equations to

$$\frac{A'(\underline{\alpha})}{A'(\overline{\alpha})} \geq \frac{-B'(\underline{\alpha})}{-B'(\overline{\alpha})}. \tag{20}$$

Since $A'' > 0$ and $B'' > 0$, we have $A'(\overline{\alpha}) > A'(\underline{\alpha})$ and $B'(\overline{\alpha}) > B'(\underline{\alpha})$. Thus (20) is not fulfilled, and there is a contradiction. It follows that there cannot be an interval without mass on $(0, 1)$.