

Online Appendix for “Performance Evaluation and Financial Market Runs”

Existence of An Interior Degree of Delegation

We first derive a condition for optimal delegation being strictly positive ($n^* > 0$). Since the (marginal) gains from delegation are falling in the mass of delegating investors, a necessary and sufficient condition for this is that the gains from having only a few (technically: a mass of zero) investors delegating are positive.

When only a few investors delegate, we have $c = 0$. Differentiating welfare (equation (17)) with respect to n we obtain at $n = 0$:

$$\left. \frac{\partial W(\hat{e})}{\partial n} \right|_{n=0} = \bar{b} + \frac{\hat{e}-1}{2} - K(\hat{e}). \quad (38)$$

Delegation will thus be optimal whenever there is a performance contract that can induce an effort level \hat{e} for which $\bar{b} + \frac{\hat{e}-1}{2} - K(\hat{e}) > 0$. Note from equation (14) that the planner can induce any (positive) effort level through an appropriate choice of the contract parameters (for example, by setting a sufficiently high). Thus, in order for delegation to be optimal we need to have that there exists an effort level e at which $\bar{b} + \frac{e-1}{2} - K(e) > 0$. This will obviously be the case whenever we have $\bar{b} + \frac{e^*-1}{2} - K(e^*) > 0$ at the e^* that maximizes $\bar{b} + \frac{e-1}{2} - K(e)$. This effort e^* is determined by

$$\frac{\partial^2 W(e)}{\partial n \cdot \partial e} = \frac{1}{2} - (z-1)ke^{*z-1} = 0. \quad (39)$$

Solving for e^* and inserting into $\bar{b} + \frac{e^*-1}{2} - K(e^*) > 0$ gives

$$k < \frac{1}{2(z-1)} \left(\frac{z}{2(z-1)(\frac{1}{2} - \bar{b})} \right)^{z-1}. \quad (40)$$

The right-hand side of this inequality is strictly larger than zero (since $z > 1$ and $\bar{b} < \frac{1}{2}$). It follows that for sufficiently small k optimal delegation is always positive.

Next, observe that the benefits from delegation (equation (17)) become negative for large enough n (in particular, the term $-r\frac{c}{2} = -r\frac{n\gamma\sigma^2}{2}$ will go to minus infinity as n becomes large). It follows that when \bar{n} is sufficiently large, complete delegation in the

economy ($n = \bar{n}$) will not be optimal. An interior degree of delegation can thus exist in the economy.

Other Incentive Devices

Our analysis has focused on the dismissal threat at an intermediate date as the source of managerial discipline. Disciplining managers directly by restricting their set of actions is not feasible in our economy due to the unobservability of effort and run decisions. Alternatively, discipline may also be provided in the form of performance fees based on date 1 or date 2 market prices. Such fees may make a difference for the results since they do not restrict managerial incentives to exceeding the firing threshold (under the contract considered in our model the manager does not benefit from better performance once he is above the threshold). However, it can be shown that such performance fees still create runs. The reason is simple: in order to induce effort, performance fees have to be increasing in a manager's absolute and/or relative performance. This in turn induces managers to participate in runs as abstaining from a run lowers performance (in either absolute or relative terms) and reduces fees.

In order to understand the impact of performance fees in our setting, consider a manager who receives a monetary compensation b_i which may depend on his absolute and/or his relative performance, that is we have $b_i = b_i(p_i, p)$.¹ Suppose that the compensation is (strictly) increasing in his own performance: $\partial b_i / \partial p_i > 0$ (in particular, one could make the manager the residual claimant at date 1 or 2, which would imply $\partial b_i / \partial p_i = 1$). The manager would then lose compensation if he abstains from a run regardless of the level of fundamentals. This is because when he joins a run the (absolute) performance of his fund will be larger than if he does not run. As a result the fear of others running would always make the manager run in the risk-dominant equilibrium. The same holds true if managerial compensation is increasing in relative performance ($\partial b_i / \partial (p_i - p) > 0$) because abstaining from a run lowers a manager's performance compared to other managers who

¹It makes no difference whether compensation is based on date 1 or 2 prices, see the discussion in the next section of this appendix.

are running, and thus also his compensation. Incentives which are (strictly) increasing in performance are thus very susceptible to runs and may hence not be optimal.

The following proposition shows that in fact any performance fee schedule (even if only weakly increasing in performance) that induce desirable levels of effort cannot resolve the run-problem:

Proposition 7 *Any optimal (symmetric) performance-fee schedule induces runs.*

Proof. *We assume managerial compensation of the form $b(p_i, p)$ with b weakly increasing in p_i , and weakly decreasing in p . Suppose that no runs ever take place. In that case prices always reflect fundamentals: $p_i = e_i v$ and $p = e v$, and we have that $b = b(e_i v, e v)$. The first order condition for effort is then*

$$\int_0^1 v b_1(e_i v, e v) dv = k z e_i^{z-1}, \quad (41)$$

where b_1 denotes the derivative with respect to the first argument. For any optimal contract we need that $\tilde{e}_i > 0$ since otherwise the economy would be strictly better off if investor i were to invest directly rather than delegate. Since in equilibrium we have $\tilde{e}_i = \tilde{e}$, we thus obtain from above equation

$$\int_0^1 v b_1(\tilde{e} v, \tilde{e} v) dv > 0. \quad (42)$$

It follows that there is a set of $x \in [0, \tilde{e}]$ that has a positive mass and for which we have $b(x, x) > b(x - \frac{c}{2}, x)$. Take an arbitrary x from this set and consider the shock $v = \frac{x + \frac{c}{2}}{\tilde{e}}$. For this shock the unique equilibrium under flat beliefs is for all managers to run. To see this, consider manager i and suppose first that all other managers are not running. Since in this case we have $p_i = \tilde{e}_i v$ and $p = \tilde{e} v$, the manager's remuneration does not depend on whether he runs or not. Suppose next that all other managers are running. The pay-off difference from running is then $b(\tilde{e} v - \frac{c}{2}, \tilde{e} v - \frac{c}{2}) - b(\tilde{e} v - c, \tilde{e} v - \frac{c}{2})$. This is because by abstaining from a run the manager is evaluated at post-run prices that incur a discount of c , while if he runs he only suffers the average in-run discount $\frac{c}{2}$. Since $\tilde{e} v - \frac{c}{2} = x$, this is equal to $b(x, x) - b(x - \frac{c}{2}, x)$, which is strictly positive by the property of x . Hence it is optimal for the manager to run. It follows that the equilibrium is that all managers run.

Since there is a positive mass of such x , it also follows that there is a positive probability of all managers running. ■

By assuming that managers only consume at date 2, our analysis has also ruled out remunerating managers through the assets' final pay-offs at date 3. This is consistent with the observation that in practice fund managers are rarely incentivized through long-term pay-offs, such as dividends. An obvious reason for this is that it typically takes a substantial amount of time for such pay-offs to materialize in full, while the horizon of fund managers is relatively short-term. Another reason is that the final value of an asset will be subject to additional noise outside the manager's control and is hence less informative about his effort choice. This reduces the effectiveness of incentives based on these pay-offs. When the additional noise is sufficiently large, such incentives become inferior to using the firing-threat at date 1, even though the latter induces costs by causing runs.

In order to understand the impact of incentivizing fund managers through date-3 pay-offs, we modify the baseline model as follows. First, we assume that managers can also consume at date 3. Moreover, we assume that the final-date value of an asset is also subject to fund-specific uncertainty. The reason for this modification is that the influence of aggregate shocks can be completely eliminated by using RPE. Thus we need idiosyncratic uncertainty in order to make performance evaluation noisy. In particular, we assume that the fund's final date value is now $e_i \tilde{v} + \tilde{\varepsilon} + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i$ is an additional fund-specific shock. This shock is uniformly distributed on $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The shock has thus zero mean and a density of $\phi = \frac{1}{2\bar{\varepsilon}}$. The shock is also assumed to be independently distributed across funds.

By the law of large numbers, the fund-specific shocks then cancel out in the aggregate. It follows that this risk is not priced by the market maker at date 1. As a consequence, date 1 and 2 prices are unchanged and our analysis of optimal performance evaluation through market prices in the previous section does not change. In addition we have:

Proposition 8 *When idiosyncratic noise is sufficiently high (that is, when $\bar{\varepsilon}$ large), it is not optimal to incentivize the manager through the final-date asset value.*

Proof. *Denote the level of effort to be induced by $\bar{\varepsilon}$. As there is now no cost of using RPE based on the final-date value, optimal compensation will use complete RPE. Hence,*

compensation will be conditional on the difference between the liquidation value of fund i , $e_i\tilde{v} + \tilde{\varepsilon} + \tilde{\varepsilon}_i$, and the average liquidation value of all funds, which is $\bar{e}\tilde{v} + \tilde{\varepsilon}$ in equilibrium. Thus, a manager is fired if his performance $(e_i - \bar{e})\tilde{v} + \tilde{\varepsilon}_i$ does not exceed a , or if

$$(e_i - \bar{e})\tilde{v} + \tilde{\varepsilon}_i \leq a. \quad (43)$$

Otherwise he is retained and obtains benefits $b \leq \bar{b}$. Rearranging for $\tilde{\varepsilon}_i$, we obtain that for a given \tilde{v} the manager is retained iff $\tilde{\varepsilon}_i(\tilde{v}) > a - (e_i - \bar{e})\tilde{v}$. Hence, expected managerial pay-off is

$$\int_0^1 \left(\int_{a-(e_i-\bar{e})v}^{\bar{\varepsilon}} b\phi(\bar{\varepsilon})d\varepsilon \right) dv. \quad (44)$$

The manager's first order condition is then

$$\int_0^1 bv\phi(\bar{\varepsilon})dv = kze_i^{z-1}. \quad (45)$$

Rearranging for $e_i = e$ gives $e = \left(\frac{b}{4\bar{\varepsilon}kz}\right)^{\frac{1}{z-1}}$. Noting that this is increasing in b we can set b to its maximum feasible level \bar{b} . We hence obtain for maximum effort that can be induced:

$$\bar{e} = \left(\frac{\bar{b}}{4\bar{\varepsilon}kz}\right)^{\frac{1}{z-1}}. \quad (46)$$

This expression becomes arbitrarily small for large $\bar{\varepsilon}$. By contrast, the maximum effort that can be induced by motivating the manager through date 1 performance contracts does not depend on $\bar{\varepsilon}$ and is unbounded (consider for example setting d close to 1 in equation (14)). Thus, whenever inducing a large amount of effort is socially desirable (for example, because effort costs are low), using the final-date value as an incentive scheme is not preferable when $\bar{\varepsilon}$ is large. This can alternatively be seen directly by observing that the maximum effort approaches zero in equation (46) when $\bar{\varepsilon}$ becomes large, in which case delegation would be dominated by direct investment. ■

Separation of Runs and Performance Evaluation

The date of arrival of the (intermediate) shock and the date of the performance evaluation coincide in our setup. This seems restrictive. What happens if the date of performance evaluation is some time after the shock arrives?

Suppose that the manager can be fired at date 2 rather than at date 1 (and that this is still costly for the manager). Assume that managers can, besides selling assets at date 1, also sell assets at date 2 (before their performance evaluation). Consider a situation where managers would run in our baseline model (in which shock and performance evaluation coincide at date 1). Now suppose that there is no run at date 1 in the modified setup. Clearly, there would then be a run at date 2, since no new information arrives between date 1 and 2 and thus the situation at date 2 is the same as in the baseline model at date 1. However, this cannot be an equilibrium: managers would then be (individually) better off selling at date 1. Thus, there will be a run at date 1. Hence, when shock and performance evaluation are separated, the threat of a run at the performance evaluation date can cause managers to preempt the run.

Asymmetric Performance Contracts and Specialization in Different Assets

We have solved for the optimal performance contract, restricting it to be symmetric. One may conjecture that by allowing contracts to differ among fund managers, the problem of runs can be avoided. This is because managers will then tend to be close to the firing-threshold at different times and hence there may no longer be a threat of a collective run. However, this is not the case. The intuition is as follows. In order for a delegated investment to add value, each manager needs to be incentivized such that he exerts (positive) effort. As argued earlier, this requires that a manager is fired following negative performance. For a sufficiently low realization of the shock v this implies that all managers will be fired regardless of whether their performance contracts are symmetric or not. In such cases managers have an incentive to collectively run in order to avoid underperformance. Hence, runs still occur.

In order to show this formally, we relax the assumption that the planner has to dictate identical contracts to managers, that is, the parameters a , b and d are now allowed to differ across delegation relationships. We obtain the following proposition.

Proposition 9 *The economy (optimally) suffers strictly positive costs due to runs even when performance contracts are allowed to be asymmetric.*

Proof. *Consider an arbitrary distribution of delegation contracts among delegating investors. This distribution can be characterized by a density $h(a, b, d)$ with $h \geq 0$ and $\int_0^\infty (\int_0^{\bar{b}} (\int_0^1 h(a, b, d) dd) db) da = n$. Under these contracts, each individual fund manager chooses an effort $\tilde{e}_i > 0$. If this were not the case, the economy would be strictly better off by letting this manager's investor invest directly (the expected pay-off from investment would then be at least as high and additionally the economy would save the managers' effort costs and, potentially, also run costs). We show that for any such allocation there is a strictly positive probability that all managers run collectively, thus incurring costs $\frac{c}{2}$ per delegating investor.*

The condition that manager i is fired is given by (analogous to equation (11)) $e_i v - c(v)/2 - d_i(ev - c(v)/2) \leq a_i$, where $c(v) = m(v)\gamma\sigma^2$ and $m(v)$ is the mass of other managers running when shock v is realized. An individual manager's firing threshold is then given by $f_i(m) = \frac{a_i + (1-d_i)\frac{m\gamma\sigma^2}{2}}{e_i - d_i e}$. His resulting effort choice is given by (analogous to equation (13))

$$\frac{(a_i + (1 - d_i)\frac{m\gamma\sigma^2}{2})b_i}{(\hat{e}_i - d_i e)^2} = zk\hat{e}_i^{z-1}. \quad (47)$$

Rearranging gives $a_i = zk\hat{e}_i^{z-1}(\hat{e}_i - d_i e)^2/b_i - (1 - d_i)\frac{m\gamma\sigma^2}{2}$. Using this equation to eliminate a_i in the firing condition, we obtain after rearranging

$$v \leq \frac{zk\hat{e}_i^{z-1}(\hat{e}_i - d_i e)}{b_i}. \quad (48)$$

The right-hand side of this inequality is strictly larger than zero since $\hat{e}_i > 0$ and $(\hat{e}_i - d_i e)/b_i > 0$ (the latter is because otherwise, as shown by the effort condition (47), no positive effort is induced). Hence, for sufficiently small (but strictly positive) v the inequality is always fulfilled and the manager is fired.

Define with \hat{v} the largest v at which above condition is fulfilled for all managers. From the above considerations we have that $\hat{v} > 0$. For $v \in [0, \hat{v}]$ we hence have that all managers will be fired. The equilibrium under flat beliefs is then for all managers to run. This is because when all managers run, it is individually optimal for each manager to join since

he can then increase his performance conditional on being fired. The costs induced for each investor are then $\frac{c}{2}$ per run, hence total costs in the economy are at least $n\widehat{v}\frac{c}{2}$. ■

Similarly to asymmetric contracts, run problems may also be mitigated if managers could select different assets in order to reduce the risk of jointly experiencing low performance. This effect does not arise in our setup since assets (conditional on effort) are only subject to aggregate uncertainty. Consider, to the contrary, first a situation where assets are after date 1 also subject to idiosyncratic uncertainty (on top of aggregate uncertainty). This would give managers an incentive to invest in different assets at date 0 such that they hold (collectively) a diversified portfolio. In a run the market maker, who purchases from many investors, would then not be subject to idiosyncratic risk and hence would not charge a premium for it. We would then obtain the same demand curve as in Section 2 and the analysis of runs would be unchanged (see also footnote 15 in the paper).

Consider alternatively that assets are hit by an idiosyncratic shock (on top of the aggregate shock) at date 1 (the previous paragraph considered shocks that occur after that date). In particular, suppose that each asset is hit with equal probability by an additional shock of s or $-s$. If managers then specialize in different assets at date 0, they reduce the likelihood that many of them jointly incur low performance. In this case, lucky managers (which have received a positive shock on their assets) may not to be fired even when the aggregate shock is relatively unfavorable. The unlucky managers, however, may become more likely to be fired since they are now always underperforming relative to their peers (of which half have received a good shock). Hence managers do not necessarily benefit from specialization and may thus still prefer all to invest in the same asset. In any case, even if managers choose different assets, runs cannot be avoided since unlucky managers still face the threat of being fired and hence have an incentive to run.

Another reason why managers may specialize in different assets is in order to reduce coordination problems in each asset market. Suppose that there are many assets, each of which is served by a fully specialized market maker. When specializing each in one (different) asset market, managers then no longer interact with the managers that have invested in other assets. This reduces the coordination problems that are the source of

runs. At the same time, however, it also increases the selling pressure created by a manager present in an asset class (each manager will sell more of the asset when he is fully specialized in it). We show in the following that as long as the number of assets is small relative to the number of managers, the second effect neutralizes the first. The overall expected run costs then do not change when managers specialize. Run costs, however, might be reduced if managers could specialize in a large number of assets such that there are, for instance, only a few managers per asset class. In this case, coordination failures indeed become less likely (and in the extreme case of one manager per asset class they can obviously be completely avoided).

To analyze this case, suppose that there are now m (identical) assets instead of the single asset in the baseline model. Each of the assets is served by one market maker (who is fully specialized in this asset). Suppose first that fund managers perfectly specialize in (different) assets. Given a mass of fund managers n , this implies that each asset is chosen by a mass of $\frac{n}{m}$ investors. The discount when all managers of an asset class are running is hence $c = \frac{n}{m}\gamma\sigma^2$ (instead of $c = n\gamma\sigma^2$ as in the baseline model). As there are no interactions among the various asset markets, the run condition (equation (10) with the modified run costs $c = \frac{n}{m}\gamma\sigma^2$) now applies to each asset in isolation. In addition, due to the symmetry of setup, runs across asset classes will be perfectly correlated in equilibrium. Given run costs per manager of $\frac{n}{m}\gamma\sigma^2$, the total cost of a run in the economy will be $n \cdot \frac{n}{m}\gamma\sigma^2$.

Suppose alternatively that managers do not specialize but instead spread their funds equally over all assets. They thus invest a share $\frac{1}{m}$ of their funds in each asset. Ruling out partial runs (where an individual manager runs in one market but not in another) we thus face the same situation as in the baseline model: a manager's performance depends on the total number of other managers in the economy running. When all managers run, total selling pressure per asset is then $\frac{n}{m}$ as each manager sells $\frac{1}{m}$ of an asset. Asset discounts are thus $\frac{n}{m}\gamma\sigma^2$ and are identical to the case where managers specialize. The run condition, which now applies in the aggregate and not to each market separately, is thus identical to the one of the baseline model with $c = \frac{n}{m}\gamma\sigma^2$. As just discussed, this is also the run condition for the specialization case. Hence both run cost and the run condition do not

change if one moves from a diversified to a specialized economy. Coordination problems (or, more precisely, the costs arising from coordination problems) thus do not change when managers specialize in the presence of specialized market makers.

It should be noted that this argumentation relies on the number of assets/market-makers being small relative to the number of managers. Suppose instead that there are m fund managers when there are m assets. Then, if each manager specializes in exactly one asset, there are no longer any coordination problems and runs can be entirely avoided. However, this is a less realistic scenario since in practice the number of funds is large relative to the number of investable assets. If they were all to specialize in different assets (for example, within the S&P500), there would be many fund managers per asset and hence coordination problems would still be present. In addition, specialization would also have a significant disadvantage: the loss of diversification benefits (while for the purpose of the extension we have assumed that all assets are subject to the same risk, in reality assets will be subject diversifiable idiosyncratic risk). Even in the setting of the paper (where investors are not risk-averse) a loss of diversification is costly (as we have argued earlier in our discussion on specialization in the presence of idiosyncratic uncertainty): specialization makes a manager's absolute and relative performance more volatile and hence increases the likelihood of falling below the threshold.

Furthermore, it should be kept in mind that an implicit assumption in the extension considered here is that liquidity supply is entirely asset-specific (market-makers are completely specialized and there is no arbitraging among (identical) assets). This assumption is not realistic. In practice, liquidity has an important systematic component (take for instance the 1987 stock market crash where prices on most assets fell simultaneously). When liquidity is systematic, specialization obviously does not help as then runs in one asset market will drain liquidity from another asset market, implying that coordination problems cannot be avoided by investing in different assets.

The Equilibrium Selection

We have selected an equilibrium motivated by the concept of risk-dominance. It is easy to see that there is another equilibrium in which managers never run. This is because an individual manager never benefits from running if other managers do not run (since prices will then reflect fundamentals). In this case there are obviously no costs to delegation. The (marginal) gains from delegation will then be constant, implying that an optimal allocation would either stipulate delegation for all investors or for no investor. The structure of performance evaluation (that is, how to combine absolute and relative evaluation) would then become irrelevant as in our framework its sole objective is to minimize runs.

The concept of risk dominance, however, has wide appeal in our setting since it applies to situations where agents face high uncertainty about the actions of other agents and cannot easily coordinate on the “good” (that is, the no-run) equilibrium. In particular, it is difficult to see how coordination among a large number of fund managers can be achieved given that individual selling decisions are not easy to observe. Risk-dominance also has a strong appeal since it is typically obtained when investors’ signals about fundamentals are imprecise (Carlsson and van Damme, 1993). Furthermore, sunspot driven equilibria can be interpreted in terms of risk dominance (Ennis, 2003, p.66) and learning models often converge to the risk dominant equilibrium as well (e.g., Kandori, Mailath, and Rob, 1993).²

While the risk-dominant and the no-run equilibrium are effectively polar cases, our results also apply to intermediate situations. Suppose for example that a manager believes that a fraction q ($q > 0$) of other managers is running. In that case it is easy to see that the run condition is $e_i v - qc - d(ev - q\frac{c}{2}) \leq a$ (the considerations are analogous to Proposition 1), which is the same as the run condition in the baseline model (equation (10)) when c is replaced by qc . This replacement can be carried through the entire analysis; hence all (qualitative) results continue to hold.

²For the equilibrium selection it is also not important that manager’s attach *equal* probabilities to the likelihood of other managers flipping their actions. It is easy to verify that the proof of Proposition 1 also holds for any interior probability ($p \in (0, 1)$) of managers flipping their actions.

Price Impact of Asset Sales Outside Managerial Runs

We have implicitly assumed that direct investors can wait until asset pay-offs have fully materialized. In practice, investors may have to sell assets earlier at market prices. It could be argued that this subjects them to similar problems as fund managers: investors may fear that other investors are selling (and that thus prices will be depressed) which could make them inclined to sell as well.

However, there are important differences between investors and managers. Investors have a longer horizon (which typically extends beyond the time for which a manager is employed). More importantly, investors also have more flexibility. They know that they could always postpone their asset sales and they should thus be less concerned about runs. Our assumption of investors being able to wait until the asset's pay-offs materialize is a short-cut for modelling this flexibility. Managers, by contrast, are subject to scheduled performance evaluations. Their performance contracts cannot be designed to postpone evaluation (or to treat managers more leniently) when prices are depressed due to a run, as runs cannot be easily contracted upon.

We have also assumed that managers can buy assets at date 0 without price impact, while selling at date 1 causes prices to decline. This is a short-cut for a higher urgency of selling in a run, compared to the time a manager has to build up his portfolio.³ One may imagine, for example, that managers have time up to a date $\frac{1}{2}$ to buy assets. During that time assets may continuously be supplied by noise traders (which could be seen as retiring investors in an overlapping generations framework). Managers can then spread their purchases over time and hence will be able to avoid a significant price impact. By contrast, in a run fund managers have to act very quickly in order to avoid short-term losses. For example, in the 1987 stock-market crash most of the losses were incurred within a single day. During such a short amount of time there will be no naturally offsetting supply of liquidity from noise traders, hence causing significant price declines.

³In addition, in the real world funds are not set up simultaneously and hence asset purchases are naturally spread over time.

References

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