Internet Appendix for "Systemic Liquidation Risk and the Diversity-Diversification Tradeoff"[1]

The Assumptions on the Distribution of the Asset Returns

In this appendix we derive conditions for which the density of asset returns ϕ fulfills Assumption 1. We also check whether reasonable parameterizations of the lognormal and the exponential distribution fulfill the assumption.

We first write the expected likelihood of liquidations, $E_0[\mathbf{L}_\alpha]$, in integration-form. From $v = \alpha y + (1-\alpha)x$ (equation (1)) and the fact that liquidations take place whenever $v + R < d$, we can derive the critical return $\widehat{y}(x)$ that just avoids liquidation (that is, there are liquidations whenever $y < \widehat{y}(x)$ but no liquidations when $y \geq \widehat{y}(x)$). This return is given by

$$\widehat{y}(\alpha, d) = \frac{d-R}{\alpha} - \frac{1-\alpha}{\alpha}x. \tag{20}$$

We hence have for the total likelihood of liquidations

$$E_0[\mathbf{L}_\alpha] = \int_0^{\widehat{x}(0)} \left(\int_0^{\widehat{y}(x)} \phi(x)\phi(y)dy \right) dx, \tag{21}$$

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where $\widehat{x}(0)$ $(:= \frac{d-R}{1-\alpha})$ denotes the x at which $\widehat{y}(x) = 0$. The derivative of $E_0[\mathbf{L}_\alpha]$ with respect to α is then

$$E_0[\frac{\partial \mathbf{L}_\alpha(x,y)}{\partial \alpha}] = \int_0^{\widehat{x}(0)} \frac{x-(d-R)}{\alpha^2}\phi(x)\phi(\widehat{y}(x))dx, \tag{22}$$

where we have used that $\frac{\partial \widehat{y}(x)}{\partial \alpha} = \frac{x-(d-R)}{\alpha^2}$. Define $\widetilde{\phi}$ to be the average density in (22):

$$\widetilde{\phi} := \sqrt{\frac{\int_0^{\widehat{x}(0)} \phi(x)\phi(\widehat{y}(x))dx}{\widehat{x}(0)}}. \tag{23}$$

Using $\widetilde{\phi}$ we can rewrite the effect of diversification on liquidations as

$$\begin{aligned} E_0[\frac{\partial \mathbf{L}_\alpha(x,y)}{\partial \alpha}] &= \int_0^{\widehat{x}(0)} \frac{x-(d-R)}{\alpha^2}\widetilde{\phi}^2 dx + \int_0^{\widehat{x}(0)} \frac{x-(d-R)}{\alpha^2}(\phi(x)\phi(\widehat{y}(x)) - \widetilde{\phi}^2)dx \\ &= \frac{(2\alpha-1)(d-R)^2}{2(1-\alpha)^2\alpha^2}\widetilde{\phi}^2 + \frac{1}{\alpha^2}\int_0^{\widehat{x}(0)} (x-(d-R))(\phi(x)\phi(\widehat{y}(x)) - \widetilde{\phi}^2)dx \end{aligned} \tag{24}$$

Consider $\alpha < 0.5$ (calculations for $\alpha > 0.5$ follow from symmetry of the problem). The first term is then unambiguously negative. The second term is determined by the covariance of $x-(d-R)$ and $\phi(x)\phi(\widehat{y}(x)) - \widetilde{\phi}^2$ since $\int_0^{\widehat{x}(0)}(\phi(x)\phi(\widehat{y}(x) - \widetilde{\phi}^2)dx = 0$ by the definition of $\widetilde{\phi}$. This covariance will be driven by the shape of $\phi(.)$ and can be either positive or negative. However, when ϕ is sufficiently flat (that is, if it does not vary much), the variance of $\phi(x)\phi(\widehat{y}(x)) - \widetilde{\phi}^2$ will be small and hence the second term will be small as well (when ϕ is constant (uniform distribution) the second terms is exactly zero). The first term will then dominate and we hence have $E_0[\frac{\partial \mathbf{L}_\alpha}{\partial \alpha}] < 0$ and part (i) of Assumption 1 is fulfilled. However, it is easy to show that there are also density functions for which $E_0[\frac{\partial \mathbf{L}_\alpha}{\partial \alpha}] > 0$ for at least some α.

We next consider part (ii) of Assumption 1, which is based on the second derivatives of $E_0[\mathbf{L}_{\alpha,x<,y}]$ and $E_0[\mathbf{L}_{\alpha,x>,y}]$. We focus on $E_0[\mathbf{L}_{\alpha,x<,y}]$ (results for $E_0[\mathbf{L}_{\alpha,x>,y}]$ follow from symmetry). We have for $E_0[\mathbf{L}_{\alpha,x<,y}]$ in integration form

$$E_0[\mathbf{L}_{\alpha,x<,y}] = \int_0^{d-R} \int_x^{\widehat{y}(x)} \phi(x)\phi(y)dydx. \tag{25}$$

Taking twice the derivative with respect to α we obtain

$$\begin{aligned} E_0[\frac{\partial^2 \mathbf{L}_\alpha(x,y)}{\partial^2 \alpha}] &= \frac{(d-R)^2}{\alpha^3} - \frac{2}{\alpha^3}\int_0^{d-R}(x-(d-R))(\phi(x)\phi(\widehat{y}(x)) - \phi^2)dx \\ &+ \frac{1}{\alpha^4}\int_0^{d-R}(x-(d-R))^2\phi(x)\phi'(\widehat{y}(x))dx. \end{aligned} \tag{26}$$

The first term is strictly positive. The second term depends again on the covariance of $x-(d-R)$ and $\phi(x)\phi(\widehat{y}(x)) - \phi^2$ and will hence be small if ϕ does not vary much. The sign of the third term will depend on the sign of $\phi'(\widehat{y}(x))$, which is ambiguous since it is determined by the distribution function $\phi(.)$. However, if ϕ is relatively constant, $\phi'(.)$ will be small and hence the term will tend to vanish. The first effect can then dominate, in which case we have that $E_0[\frac{\partial^2 \mathbf{L}_{\alpha,x<y}}{\partial^2 \alpha}] > 0$ (note that this is always the case when ϕ uniform). Assumption 1 (ii) is then fulfilled. However, as with part (i) of the assumption, it is also straightforward to show that there are densities for which the assumption is not met for some α.

We next numerically simulate the relevant first and second derivatives for the lognormal distribution and the exponential distribution. For the lognormal distribution we assume $d = 0.6$ and parameterize the distribution such that the expected net asset return is 10% and the likelihood of liquidation for an undiversified investor is 10% as well. The exponential distribution does not allow us to jointly match the liquidation probability and the mean asset return as it only has one parameter. We thus also vary d to match both numbers.

[INSERT FIGURE 5 HERE]

Figures 5a-d show the results. We can see that for both distributions we have that $E_0[\frac{\partial \mathbf{L}_\alpha}{\partial \alpha}] < 0$ for $\alpha < 0.5$ and $E_0[\frac{\partial \mathbf{L}_\alpha}{\partial \alpha}] > 0$ for $\alpha < 0.5$. Thus, increasing diversification always lowers the likelihood of liquidation and hence Assumption 1(i) is fulfilled. We can also see that for the lognormal distributions we have $E_0[\frac{\partial^2 \mathbf{L}_{\alpha,x<y}}{\partial^2 \alpha}] > 0$, that is the marginal benefits from diversification are declining (Assumption 1(ii)). For the exponential distribution, however, the second assumption is only fulfilled when α is not too close to zero. The reason for this is that at very polarized portfolios the marginal benefits from diversification are increasing rather than decreasing under the exponential distribution.

Many Assets

We modify the baseline model of Section II as follows. We assume that there are a continuum of assets of mass one, which are indexed by k ($k \in [0,1]$). Assets are still in infinite supply. They have the same characteristics as asset X in the baseline model, except that the date-1 return on asset k is not x but $x + z_k$, where z_k has zero mean and is iid across assets. x is thus a common factor while z_k is an asset's idiosyncratic component. We denote an investor's holdings of asset k by q_k. We have

$$\int_{k\in[0,1]} q_k dk = 1 \tag{27}$$

because of the investor's budget constraint. Investors' utility is analogous to equation (7) given by

$$U^I(\mathbf{q}) = \mu + R - (1+r)(1-w) - E_0[(R-p)\cdot \mathbf{L_q}(x,y)], \tag{28}$$

where the liquidation event $\mathbf{L_q}$ is now given by $x + \int_{k\in[0,1]} q_k z_k dk + R < d$.

We consider a proof similar to the one for Proposition 1 (page 13). We assume a situation where all investors are fully diversified and analyze the effect of an investor deviating from this allocation by investing fully in asset k.

In the complete-diversication outcome, the investor holds equal amounts of each asset. By the law of large numbers, the idiosyncratic components in his portfolio cancel out and his date-1 portfolio return is simply x. When the investor only holds asset k his date-1 return is obviously $x + z_k$. Liquidation in the two cases hence occur when $x + R < d$ and when $x + z_k + R < d$, respectively.

Analogous to equation (8) we can write the change in the investor's utility as

$$U^I(x + z_k) - U^I(x) = -E_0[(R - p)\mathbf{L}_{x+z_k}\overline{\mathbf{L}}_x] + E_0[(R - p)\overline{\mathbf{L}}_{x+z_k}\mathbf{L}_x]. \tag{29}$$

The first term, $-E_0[(R - p)\mathbf{L}_{x+z_k}\overline{\mathbf{L}}_x]$, refers to potential fire-sale losses due to new liquidations. These losses, however, are zero as in all new liquidations other investors are not liquidating, and hence we have $p = R$. The second term, $E_0[(R - p)\overline{\mathbf{L}}_{x+z_k}\mathbf{L}_x]$, gives the benefits from instances where liquidations can be avoided. As these occur only in situations where all other investors are liquidating, we have $p < R$. Hence the benefits are strictly positive. From this it follows that the investor's utility strictly increases if he deviates from full diversification. Full diversification can thus not be an equilibrium in this economy.

Endogenous Leverage

In this extension we no longer restrict the amount of debt to $d = 1 - w$. Rather, we allow investors to raise debt freely ($d \geq 0$). This has the consequence that also the total level of investment is variable: given borrowings of d an investor can invest $w + d$. In addition, we introduce fixed costs c ($c \geq 0$) of investing. These costs can be interpreted as referring to the time and effort necessary for an investor to become acquainted with investment decisions in general, and a specific investment in particular.

Given borrowing d, total funds of $w + d$, and costs c, an investor's expected utility can analogous to equation (7) be written as

$$U^I(\alpha, d) = (\mu + R)(w + d) - (1 + r)d - E_0[(R - p) \cdot \mathbf{L}_{\alpha,d}(x, y)](w + d) - c. \qquad (30)$$

Liquidations occur when the total value of the portfolio is less than the debt. The condition for this is now:

$$(v(\alpha) + R)(w + d) < d. \qquad (31)$$

$\mathbf{L}_{\alpha,d}(x, y)$ in (30) is thus the indicator function referring to the liquidation event defined by above condition. Note that the liquidation event depends on the level of borrowing d.

The first-order condition for debt is given by

$$U^{I\prime}(d) = \mu + R - E_0[(R - p) \cdot \mathbf{L}_{\alpha,d}] - (1 + r) - E_0[(R - p)\frac{\partial \mathbf{L}_{\alpha,d}(x, y)}{\partial d}](w + d) = 0. \quad (32)$$

The term $\mu + R - E_0[(R - p) \cdot \mathbf{L}_{\alpha,d}]$ represents the benefits from the additional investment that becomes possible through higher borrowing. These benefits consists of the expected fundamental return on investment, $\mu + R$, less the expected liquidation losses, $E_0[(R - p) \cdot \mathbf{L}_{\alpha,d}]$. The term $-(1+r)$ gives the cost of raising debt. Finally, the term $-E_0[(R - p)\frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}](w+d)$ is the effect of borrowing on the liquidation losses. Higher leverage can increase the instances in which an investor has to liquidate ($E_0[\frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}] > 0$) and each additional liquidation induces losses of $R-p$ per unit of investment, hence $(R-p)(w+d)$ in total. Equation (32) thus shows that at the optimal debt level the net benefits from more investment equal the associated increase in borrowing costs plus the increase in liquidation costs due to higher leverage.

We can substitute the interest rate r in (32) using the fact that in general equilibrium the expected returns of investors and creditors have to be identical. From equating equation (3) and (30), and solving for the interest rate we obtain

$$1 + r = (\mu + R) - E_0[(R - p) \cdot \mathbf{L}_{\alpha,d}] - \frac{c}{w + d}. \tag{33}$$

Inserting into the first-order condition for debt and rearranging gives

$$U^{I\prime}(d) = \frac{c}{w + d} - E_0[(R - p) \cdot \frac{\partial \mathbf{L}_{\alpha,d}(x, y)}{\partial d}](w + d) = 0. \tag{34}$$

Optimal borrowing thus equates the marginal liquidation costs from higher leverage, $E_0[(R-p) \cdot \frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}](w+d)$, with gains arising from the fact that fixed investment costs are spread over a larger amount of investment. The latter effect is given by the term $\frac{c}{w+d}$.

We first show that optimal leverage has to be strictly positive. For this note from (31) and $v(\alpha) \geq 0$ (since $x, y \geq 0$) that liquidations can only arise if $R(w+d) < d$. Rearranging for d we obtain a lower bound for d:

$$d > \frac{w}{1/R - 1}. \tag{35}$$

For $d \leq \frac{w}{1/R-1}$ we then have $E_0[(R-p) \cdot \mathbf{L}_{\alpha,d}] = 0$, and hence also $E_0[(R-p) \cdot \frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}] = 0$. Thus, for $d \leq \frac{w}{1/R-1}$, the marginal benefits from raising leverage are positive: $U^{I\prime}(d) > 0$. It follows that optimal leverage (if it exists) is strictly positive.

We next show that the net benefits from borrowing can be declining, thus allowing for an interior solution. We already know that they are (weakly) declining when (35) is not fulfilled. We suppose now that (35) is fulfilled. From $v = \alpha y + (1-\alpha)x$ (equation (1)) and equation (31), we can derive the critical return $\widehat{y}(x)$ that just avoids liquidation (that is, there are liquidations whenever $y < \widehat{y}(x)$ but no liquidations when $y \geq \widehat{y}(x)$). This return is given by

$$\widehat{y}(\alpha, d) = \frac{\frac{d}{w+d} - R}{\alpha} - \frac{1-\alpha}{\alpha} x. \tag{36}$$

We can use this to write the expected liquidation costs in integration form:

$$\begin{aligned} E_0[(R-p) \cdot \mathbf{L}_{\alpha,d}] &= \int_0^{\widehat{x}(0)} \left(\int_0^{\widehat{y}(x)} (R-p)\phi(x)\phi(y)dy \right) dx \\ &= \int_0^{\frac{1}{1-\alpha}(\frac{d}{w+d}-R)} \left(\int_0^{\frac{\frac{d}{w+d}-R}{\alpha} - \frac{1-\alpha}{\alpha}x} (R-p)\phi(x)\phi(y)dy \right) dx, \end{aligned} \tag{37}$$

where $\widehat{x}(0)$ $(= \frac{1}{1-\alpha}(\frac{d}{w+d} - R))$ denotes the x at which $\widehat{y}(x) = 0$. Differentiating with respect to d gives

$$E_0[(R-p)\frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}] = \frac{w}{\alpha(w+d)^2}\int_0^{\frac{1}{1-\alpha}(\frac{d}{w+d}-R)}(R-p(\widehat{\alpha}(x,\widehat{y}(x))))\phi(x)\phi(\widehat{y}(x))dx. \tag{38}$$

Writing this expression for an investor with $\alpha = 0.5$ yields

$$E_0[(R-p)\frac{\partial \mathbf{L}_{0.5,d}(x,y)}{\partial d}] = \frac{2w}{(w+d)^2}(R-\frac{wL}{I/2})\int_0^{2(\frac{d}{w+d}-R)}\phi(x)\phi(2(\frac{d}{w+d}-R)-x)dx, \tag{39}$$

where we have used that $G(\widehat{\alpha}(x,\widehat{y}(x))) = I - G(\widehat{\alpha}(x,\widehat{y}(x))) = \frac{I}{2}$ when $\widehat{\alpha}(x,y) = 0.5$. Expression (39) is larger than zero, confirming that higher leverage increases expected liquidation costs when $d > \frac{w}{1/R-1}$. Using the expression to substitute $E_0[(R-p)\frac{\partial \mathbf{L}_{\alpha,d}(x,y)}{\partial d}]$ in (34) and rearranging gives

$$U^{I\prime}(d) = \frac{1}{w+d}\left(c - 2w(R-\frac{wL}{I/2})\int_0^{2(\frac{d}{w+d}-R)}\phi(x)\phi(2(\frac{d}{w+d}-R)-x)dx\right) = 0. \tag{40}$$

Differentiating again with respect to d gives

$$\begin{aligned} U^{I\prime\prime}(d) &= -\frac{1}{(w+d)^2}\left(c - 2w(R-\frac{wL}{I/2})\int_0^{2(\frac{d}{w+d}-R)}\phi(x)\phi(2(\frac{d}{w+d}-R)-x)dx\right) \\ &\quad \frac{4w^2(R-\frac{wL}{I/2})}{(w+d)^3}\left(-\phi(x)\phi(0) - \int_0^{\frac{d}{w+d}-R}\phi(x)\phi'(\widehat{y}(x))dx\right). \end{aligned} \tag{41}$$

The term in the first line is zero at d^* (follows from equation (40). In the second line, the first term inside the large brackets, $-\phi(x)\phi(0)$, is strictly negative. This is the effect that arises because when leverage is large, liquidations occur often. As a result, a further increase in leverage increases liquidations more (in absolute terms), thus reducing the marginal benefits from leverage. The sign of the second term inside these brackets,

$-\int_0^{(\frac{d}{w+d}-R)} \phi(x)\phi'(\widehat{y}(x))dx$, depends on the sign of $\phi'(\widehat{y}(x))$, that is how the density of the asset return changes when the critical return $\widehat{y}$ is raised. When $\phi(.)$ is relatively flat, this term will be small (in particular, when ϕ uniform, the expression will be obviously be zero). The first term can hence dominate. It follows that the marginal benefits from borrowing can be declining, and hence an interior solution may exist.

Finally, we analyze equilibrium leverage in the economy using the parameterization of the baseline model. First, we study whether the marginal gains from borrowing are indeed declining. For this we assume fixed costs of 0.01 (1% of total invested funds) and use the same proportion of agents as in the baseline example.

[INSERT FIGURE 6 HERE]

Figure 6 depicts the marginal benefits for d and shows that the marginal gains are initially constant. This is because for small amounts of leverage there are no liquidations (the threshold d is actually at $\frac{w}{1/R-1} \approx 0.17$). We can also see that the marginal gains eventually decline rapidly and become zero at around 0.55. Thus, there exists an optimal interior degree of leverage which is here $d^* = 0.55$.

Next we solve numerically for the costs c that make it optimal for an investor to borrow exactly $d = 1 - w$ (as presumed in the baseline model). We find that these costs are 0.014. This confirms that the leverage assumed in the baseline model can also be optimal if investors can choose borrowing freely. A consequence of the fixed costs, however, is that the equilibrium shares of agents change as well (compared to the equilibrium of the baseline example) because investing is now less attractive. We find that the new equilibrium shares are $I^* = 37.9\%$, $D^* = 56.8\%$ and $L^* = 5.3\%$, which are slightly different from the ones without fixed costs ($I^* = 38.1\%$, $D^* = 57.2\%$ and $L^* = 4.7\%$).

The impact on portfolio allocations is essentially unchanged. Since the costs c are fixed, they have no direct effect on portfolio allocations. This can be seen by setting the derivative of (30) with respect to α equal to zero. Dividing by $w + d$ we obtain

$$-(R - p(G(\alpha)))E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}(x,y)}{\partial \alpha}] - (R - p(I - G(\alpha)))E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}(x,y)}{\partial \alpha}] = 0, \quad (42)$$

which is identical to equation (9) of the baseline model. There is only an indirect effect that comes through the change in agent shares (this effect is discussed in more detail in the internet appendix on "Comparative Statics"): when the number of investors declines, the number of liquidity providers increases and hence average liquidation prices increase. This reduces the costs of liquidations and, in turn, lowers the benefits from diversification (which arise because diversification reduces the likelihood of liquidations). Investors thus choose in equilibrium less diversified (that is, more heterogenous portfolios) compared to the setting with exogenous leverage and no fixed costs.

Risk-Averse Preferences

We now modify the model of Section II by assuming that investors are expected-utility maximizers with utility $u(.)$, where $u' > 0$, $u'' \leq 0$ and $u(0) = 0$. We show that Proposition 1 continues to hold, that is, the full diversification allocation is still not an equilibrium.

Recall that an investor obtains the full portfolio returns minus interest payments in the case he does not have to liquidate. Given interest rate $i(\alpha)$ we have for the utility in this case $u(v(\alpha) + R - (1+i)(1-w))$. In the case of insolvency, the investor obtains zero, hence utility is $u(0) = 0$. Total expected utility is thus given by

$$U^I(\alpha) = E_0[u(v(\alpha) + R - (1+i)(1-w)) \cdot \overline{\mathbf{L}}_\alpha]. \tag{43}$$

We consider a proof similar to the one for Proposition 1 (page 13). We assume a situation where all investors are fully diversified and analyze the effect of an investor deviating from this allocation by increasing α. From differentiating (43) with respect to α we obtain:

$$\begin{aligned} U^{I\prime}(\alpha) &= E_0[u'(v(\alpha) + R - (1+i)(1-w))(y - x - i'(\alpha)(1-w)) \cdot \overline{\mathbf{L}}_\alpha] \\ &\quad + E_0[u(v(\alpha) + R - (1+i)(1-w)) \cdot \frac{\partial \overline{\mathbf{L}}_\alpha}{\partial \alpha}]. \end{aligned} \tag{44}$$

At $\alpha = 0.5$ we have that $E_0[u(v(\alpha) + R - (1+i)(1-w)) \cdot \frac{\partial \overline{\mathbf{L}}_\alpha}{\partial \alpha}] = 0$ since $E_0[\frac{\partial \overline{\mathbf{L}}_\alpha}{\partial \alpha}] = 0$. From the symmetry of the distribution function we also have that $E_0[u'(v(\alpha) + R - (1+i)(1-w))(y - x) \cdot \overline{\mathbf{L}}_\alpha] = 0$ at $\alpha = 0.5$. The derivative at $\alpha = 0.5$ hence simplifies to

$$U^{I\prime}(0.5) = -E_0[u'(v(0.5) + R - (1+i)(1-w))i'(0.5)(1-w) \cdot \overline{\mathbf{L}}_{0.5}]. \tag{45}$$

Its sign hence depends on the sign of $i'(0.5)$, that is, the impact of an increase in α on the interest rate.

The interest rate is determined by the condition that debtors have to break even on average. Debtors receive $(1+i)(1-w)$ in the case of solvency, otherwise they receive the liquidation proceeds $v(\alpha)+p$. Their break-even condition hence writes

$$(1+i(\alpha))(1-w)E_0[\overline{\mathbf{L}}_\alpha] + E_0[(v(\alpha)+p)\cdot \mathbf{L}_\alpha] = (1+r)(1-w). \tag{46}$$

Consider now the effect of an increase in α on the expected repayments to debtors. For this partially differentiate the left-hand side of (46) with respect to α, keeping the interest rate constant. We obtain

$$(1+i)(1-w)E_0[\frac{\partial \overline{\mathbf{L}}_\alpha}{\partial \alpha}] + E_0[(y-x)\cdot \mathbf{L}_\alpha] + E_0[(v(\alpha)+p)\cdot \frac{\partial \mathbf{L}_\alpha}{\partial \alpha}]. \tag{47}$$

Evaluating at $\alpha = 0.5$ gives that the first term is zero since $E_0[\frac{\partial \overline{\mathbf{L}}_{0.5}}{\partial \alpha}] = 0$. The second terms is also zero since both assets are identically distributed. The last term can be split into the effects that occur if the investor liquidates alone and if he liquidates together with all other investors: $E_0[(v(\alpha)+p)\cdot \frac{\partial \mathbf{L}_\alpha}{\partial \alpha}\overline{\mathbf{L}}_{0.5} + (v(\alpha)+p)\cdot \frac{\partial \mathbf{L}_\alpha}{\partial \alpha}\mathbf{L}_{0.5}]$. As liquidation prices are R in the former case, while $\frac{wL}{I}$ in the latter we obtain at $\alpha = 0.5$:

$$E_0[(v(0.5)+R)\cdot \frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\overline{\mathbf{L}}_{0.5} + (v(0.5)+\frac{wL}{I})\cdot \frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\mathbf{L}_{0.5}]. \tag{48}$$

Since $R > \frac{wL}{I}$ and $E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\overline{\mathbf{L}}_{0.5}] = -E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\mathbf{L}_{0.5}]$ (because of $E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\overline{\mathbf{L}}_{0.5}] + E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\mathbf{L}_{0.5}] = E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}] = 0$) it follows with $E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial \alpha}\overline{\mathbf{L}}_{0.5}] > 0$ that this derivative is positive. Hence the repayment to debtors increases, which means that the interest rate can be reduced:

$i'(\alpha) < 0$. From (45) it then follows that the investor's utility improves, contradicting the assumption of an equilibrium.

Comparative Statics Analysis

We consider the impact of variations in the key parameters of our model (d, μ, σ^2, R) on the equilibrium choices of agents. The latter can be summarized by the agent shares (I^*, L^*, C^*) and the distribution of portfolios $G^*(\alpha)$. In order to isolate effects on the distribution that arise because the total number of investors varies, we focus on the normalized portfolio distribution $\widetilde{G}(\alpha) = \frac{G(\alpha)}{I}$ instead of $G(\alpha)$.

We first analyze the impact on the agent shares. In general equilibrium the expected returns for all agents are equalized. From equating the returns of investors and creditors (equation (3) and (7)), and solving for the return on debt we obtain

$$r = \mu + R - 1 - E_0[(R - p) \cdot \mathbf{L}_\alpha]. \tag{49}$$

We can use this equation to replace r in the expected return of an investor (equation (7)). For an investor who has chosen $\alpha = 0.5$ this yields:

$$U^I = (\mu + R - E_0[(R - p) \cdot \mathbf{L}_{0.5}])w. \tag{50}$$

Setting this expression equal to the expected return for liquidity holders (equation 4) we obtain the following equilibrium condition:

$$\mu + R - 1 - E_0[(R - p) \cdot \mathbf{L}_{0.5}] - E_0[\frac{R}{p}] = 0 \tag{51}$$

Condition (51) states that the net expected fundamental return on a portfolio, $\mu + R - 1$, minus the expected liquidation losses from holding the portfolio, $E_0[(R-p) \cdot \mathbf{L}_{0.5}]$, have to be equal to the return from holding one unit of liquidity, $E_0[\frac{R}{p}]$.

This equation can be simplified by using the fact that the combined gains of all liquidity providers from purchasing assets have to equal the combined losses from asset liquidations by investors (as fire-sales are a zero-sum event). Recalling that in equilibrium the expected losses are the same for all investors, the combined losses can simply be expressed as the losses for an investor who has chosen $\alpha = 0.5$, multiplied by the number of investors in the economy, I. Given that the gains are $(E_0[\frac{R}{p}] - 1)w$ for a liquidity provider and that there are in total a mass of L liquidity providers, this condition writes in expectation form:

$$E_0[(R - p)\mathbf{L}_{0.5}]I = (E_0[\frac{R}{p}] - 1)wL. \tag{52}$$

We can use this equation to substitute $E_0[\frac{R}{p}]$ in (51). Using in addition that

$$I = w(1 - L) \tag{53}$$

(which follows from combining the market clearing condition for debt, $(1 - w)I = wC$, with $L + I + C = 1$), we can rewrite the equilibrium condition (51) as

$$\mu + R - 1 = \frac{1}{L} E_0[(R - p)\mathbf{L}_{0.5}]. \tag{54}$$

Proposition (11) derives next the impact of changes in the model's parameters on the equilibrium shares of the three types of agents.

Proposition 11 *i) An increase in leverage d (a reduction in wealth w) lowers the share of investors I^* and increases the share of liquidity providers L^*. The impact on the share of creditors C^* is ambiguous.*

ii) An increase in the date-1 expected asset pay-off μ (by means of transforming the density function ϕ in a first-order stochastic dominant way) increases the share of investors I^* and creditors C^*, and lowers the share of liquidity providers L^*.

iii) An increase in the date-1 asset variance σ^2 (through a mean preserving spread of the density function ϕ) lowers the share of investors I^* and creditors C^*, and increases the share of liquidity providers L^*

iv) An increase in the date-2 asset pay-off R has an ambiguous impact on the share of investors I^*, creditors C^* and liquidity providers L^*.

Proof. *i) Consider condition (54). The increase in d (reduction in w) increases the likelihood of liquidations: $E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial d}] > 0$. This raises the expected losses from liquidations: $\frac{\partial E_0[(R-p)\mathbf{L}_{0.5}]}{\partial d} = (R - \frac{wL}{I\widetilde{G}(0.5)})E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial d}] > 0$. The right hand side of (54) thus increases. In order to restore equilibrium, L has to adjust such that the right hand side falls (as the left hand side is fixed). Noting that the right hand side is strictly decreasing in the share of liquidity providers L (since $p'(L) > 0$ whenever there are liquidations), this implies that that the equilibrium share of liquidity providers L^* has to increase. Since $I = w(1-L)$ (equation 53) and both w and $1-L$ fall, it follows that the share of investors I^* has to decline. On the share of creditors ($C = I(1-w)$) the are two offsetting effects: there are less investors (and hence there is a lower demand for borrowing) but at the same time each investor borrows more. Overall borrowing, and hence also the number of creditors, are thus undetermined.*

ii) An increase in μ increases the left hand side of (54) and reduces the right hand side. The latter is because an increase in μ makes low asset pay-offs less likely and thus reduces the expected liquidation costs, $E_0[(R-p)\cdot \mathbf{L}_{0.5}]$ (this follows from first-order stochastic dominance and the fact that p is (weakly) increasing in x and y). In order *to restore equilibrium, liquidation prices thus have to fall. This implies that the mass of liquidity providers L^* has to fall. The equilibrium mass of investors I^* thus increases, and hence also the mass of creditors C^* as total borrowing increases.*

iii) An increase in σ^2 by means of a mean preserving spread of the density function affects the right hand side of (54) through the expected costs from liquidations, $E_0[(R-p)\mathbf{L}_{0.5}]$. In particular, since $\mu + R - 1 > 0$ and hence $\mu + R > d$ we have that a mean preserving spread raises expected liquidation costs. The right hand side of (54) thus increases. The mass of liquidity providers L^* consequently has to increase in order to restore equilibrium. Since investors and creditors exist in fixed proportions, it follows that their masses I^* and C^* fall.

(iv) The impact of an increase in R on the difference between the left hand side and the right hand side of (54) is

$$1 - \frac{1}{L}E_0[\mathbf{L}_{0.5}] - \frac{1}{L}(R - \frac{2L}{1-L})E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial R}], \tag{55}$$

where we have used equation (53) and $\widetilde{G}(0.5) = 0.5$. Note that the last term, $-\frac{1}{L}(R - \frac{2L}{1-L})E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial R}]$, is positive since $E_0[\frac{\partial \mathbf{L}_{0.5}}{\partial R}] < 0$ (that is, an increase in R lowers the likelihood of liquidations). Expression (55) can be either positive or negative. For example, when L is close to 1 the first term outweighs the second (since $E_0[\mathbf{L}_{0.5} < 1]$) and hence the overall expression is positive. However, if L is small, the second and third term will dominate. If in addition the probability of liquidations is high ($E_0[\mathbf{L}_{0.5}]$ large) but the impact of changes in R on the probability of liquidations is small (for example, because the density ϕ of the x and y for which $v(0.5) + R = d$ is small), the expression can become negative overall. The impact of changes in R on the agent shares is hence undetermined.

■

The intuition for these results is the following. A higher d, a lower μ or a higher σ^2 each mean that investing becomes less attractive compared to holding liquidity. Thus, the equilibrium share of liquidity providers increases and the number of agents who become investors falls. The share of creditors itself mimics the one of investors as long as the borrowing per investor, d, is constant. In the case of increasing d, the impact on the amount of creditors is undetermined since the reduction in the demand for borrowing due to the lower number of investors is offset by a higher borrowing per investor.

The proposition also shows that the impact of a change in the date-2 pay-off R differs from a change in the mean of the date-1 return μ. The reason for this is that a lower R reduces the expected returns of both investors and liquidity providers. The latter is because liquidity providers profit as well from date-2 returns when they have purchased assets at date 1. A reduction in R thus does not necessarily lower the relative attractiveness of investment (and may well increase it). Hence the impact on the number of investors, creditors and liquidity providers cannot be generally determined in this case.

We now study the impact of changes in the parameters on investors' portfolio choices. The latter are implicitly determined by $U^{I\prime}(\alpha) = 0$, where $U^{I\prime}(\alpha)$ is given by equation (A2) on page 46. Using $I = w(1-L)$ (equation 53) and $\widetilde{G}(\alpha) = \frac{G(\alpha)}{I}$ we obtain:

$$U^{I\prime}(\alpha) = -(R - \frac{L}{1-L}\frac{1}{\widetilde{G}(\alpha)})E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] - (R - \frac{L}{1-L}\frac{1}{1-\widetilde{G}(\alpha)})E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] = 0. \quad (56)$$

Condition (56) pins down the distribution of investors $\widetilde{G}(\alpha)$ as a function of the date-2 pay-off R, the amount of liquidity providers L, and the impact of diversification on the liquidation probabilities, $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]$. The latter will themselves depend on the distribution function ϕ (and thus on μ and σ^2) but also on R and d.

We define next a notion of diversity in an economy's portfolio distribution.

Definition 3 *A normalized portfolio distribution $\widetilde{G}^A(\alpha)$ is said to be* <u>*more diverse*</u> *than $\widetilde{G}^B(\alpha)$ iff $\widetilde{G}^A(\alpha) > \widetilde{G}^B(\alpha)$ for $\alpha < 0.5$ and $\widetilde{G}^A(\alpha) < \widetilde{G}^B(\alpha)$ for $\alpha > 0.5$.*

A portfolio allocation is thus considered more diverse when at each α the proportion of investors choosing more polarized portfolios is higher. Note that higher diversity implies that portfolios are less diversified.

Before turning to general equilibrium analysis, the following lemma first summarizes the partial effect (that is, keeping agent shares constant) of variations in R, L and $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ on portfolio diversity.

Lemma 1 *For given agent shares, the (partial) effect of*

i) an increase in R is to reduce portfolio diversity;

ii) an increase in L is to increase portfolio diversity;

iii) an increase in $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ is to reduce portfolio diversity.

Proof. *We proof the lemma for $\alpha < 0.5$ (the proofs for $\alpha > 0.5$ follow from symmetry). i) Recall that $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] < 0$, $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] > 0$ and that $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] + E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] < 0$ since diversification (higher α) reduces the overall likelihood of liquidations. It follows that an increase in R increases $U^{I\prime}(\alpha)$ in (56). The partial derivative of $U^{I\prime}(\alpha)$ with respect to $\widetilde{G}(\alpha)$ is $\frac{\partial U^{I\prime}(\alpha)}{\partial \widetilde{G}(\alpha)} = \frac{L}{1-L}(-\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{\widetilde{G}(\alpha)^2} + \frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}{(1-\widetilde{G}(\alpha))^2})$. Since $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] < 0$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] > 0$, it follows that $\frac{\partial U^{I\prime}(\alpha)}{\partial \widetilde{G}(\alpha)} > 0$. Hence, $\widetilde{G}(\alpha)$ has to decline in order to restore equilibrium. Diversity thus falls.*

ii) The (partial) impact of an increase in L on $U^{I\prime}(\alpha)$ is given by $\frac{\partial U^{I\prime}(\alpha)}{\partial L} = \frac{1}{(1-L)^2}(\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{\widetilde{G}(\alpha)} + \frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}{1-\widetilde{G}(\alpha)})$. We thus have $\frac{\partial U^{I\prime}(\alpha)}{\partial L} < 0$ since $\widetilde{G}(\alpha) < 0.5$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] + E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] < 0$.

Noting that this is the opposite of the effect in i), it follows that $\widetilde{G}(\alpha)$ increases and hence diversity rises.

iii) If $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ *increases, $U^{I\prime}(\alpha)$ in (56) raises above zero (to see this recall that recall that $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}] < 0$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}] > 0$). Thus, analogous to i) it follows that $\widetilde{G}(\alpha)$ has to fall. Diversity thus falls.* ■

The intuition behind these results is the following. An increase in R increases the cost of liquidations (regardless of an investor's portfolio choice) by widening the gap between the date-2 asset value R and its date-1 fire-sale price p (which is independent of R). Diversification, which reduces the likelihood of liquidations, becomes more beneficial as a result. This, in turn, makes it optimal for investors to reduce diversity in their portfolios. When the amount of liquidity providers L increases, the costs of liquidating a portfolio falls as liquidation prices will be higher. This lowers the benefits from diversification, causing investors to choose more diverse portfolios. Finally, when $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ increases, diversification (an increase in α) becomes more effective in terms of reducing the likelihood of liquidations. This makes diversification more attractive, and investors choose hence a lower degree of diversity.

We are now ready to analyze the impact of the various parameters on diversity in general equilibrium.

Increase in leverage d. As previously discussed, higher leverage makes investment less attractive and thus increases the share of liquidity providers L in the economy (part i) of Proposition 11). This lowers the costs of liquidations for investors and hence induces them to lower diversification, that is, to increase diversity (part ii) of Lemma 1). A change in leverage may also impact diversity by affecting $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$, which measures the benefits of diversification in terms of reducing the likelihood of liquidations. However, the impact on these benefits is ambiguous. This is because an increase in d increases the return realizations for which liquidations take place, which can affect both $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]$. The impact on the ratio then depends on α and on the distribution function $\phi(.)$ (for the special case where ϕ is constant, it is easy to show that the effects exactly cancel out).

As long as any effect coming through $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ does not neutralize the effects coming through the increase in liquidity, an increase in d should thus increase diversity. Figure 3a illustrates the effect of a change in d numerically. For this we use the parameters of the example in Section III and consider three different values for d $(0.5, 0.6, 0.7)$. It can be seen that an increase in d indeed increases diversity: for all α it lowers the share of more diversified investors. The effect is also quite significant, suggesting that variations in investors' liquidation intensities (variations in d in our model) can have a potentially important effect on their optimal portfolio allocations.

Increase in the mean of the date-2 pay-off μ. By increasing the attractiveness of investment, this results in a lower share of liquidity providers L (part ii) of Proposition 11). This, in turn, increases the costs of liquidations. Investors thus give up diversity in order to achieve a better diversification (part ii) of Lemma 1). The potential impact coming through a change in $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ is again ambiguous. It turns out that changing μ by modifying the density ϕ can change $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]$ in relatively arbitrary ways. The effect on the ratio can thus not be generally determined (this is even the case if ϕ is transformed in a first order stochastic dominant way). We thus again resort to numerical results. For this we have chosen values of μ that produce expected net asset pay-offs of 5%, 10% and 15%. Figure 3b shows that the overall effect of an increase in μ is to reduce diversity. The effect coming through higher liquidity thus seems to outweigh any potentially neutralizing effect through $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$.

Increase in the variance of the asset pay-offs σ^2. This will lower the attractiveness of investment and cause an increase in the number of liquidity providers L (part iii) of Proposition 11). This, in turn, reduces the costs of liquidations, allowing investors to lower diversification and to achieve a higher degree of diversity (part ii) of Lemma 1). The effect on $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$ is once again ambiguous as changes in the densities can have arbitrary consequences for both $E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]$ and $E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]$. Figure 3c shows the effect of variations in σ^2 on portfolio diversity numerically. For this we have chosen values of σ^2 and μ that produce an expected likelihood of liquidation for an undiversified investor of 5%, 10% and 15%, respectively, but leave the mean asset pay-offs constant at 10%. As the figure shows, an increase in asset variance increases diversity. The effect coming through higher liquidity thus seems to outweigh any potentially neutralizing effect through $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$.

Increase in the date-2 asset pay-off R. As we can see from equation (56), a higher R raises the costs of liquidations by increasing the fundamental value of liquidated assets but leaving unaffected their liquidation price. As a consequence, it is optimal to have more diversification; diversity is hence reduced (part i) of Lemma 1). The effect of R on diversity through changes in the agent shares is undetermined, as previously discussed (part iv) of Proposition 6). The same is true for any effect coming through $\left|\frac{E_0[\frac{\partial \mathbf{L}_{\alpha,x<y}}{\partial \alpha}]}{E_0[\frac{\partial \mathbf{L}_{\alpha,x>y}}{\partial \alpha}]}\right|$. An increase in R lowers the likelihood of liquidations but from this it does not follow how the changes in liquidation probabilities, or the ratio of changes, is affected. Figure 3d depicts the equilibrium portfolio distributions for $R = 0.25, 0.3, 0.35$, showing that a higher R leads to less diversity. The direct effect of R on the liquidation costs and thus the desirability of diversification hence seems to outweigh any other, potentially neutralizing, effects.

Calculating the Output Loss in the Economy

Total expected date-2 output in the baseline economy is given by the expected asset pay-offs plus the liquidity holdings. Given that total investment in assets is $(1-L)w$ and total liquidity holdings are Lw, we thus have for expected output

$$W = (R+\mu)(1-L)w + Lw. \tag{57}$$

Since assets have an expected return higher than one (and thus more than the return to holding liquidity), expected output is maximized when the entire wealth of the economy is invested in assets. First best welfare is thus given by

$$W^* = (R+\mu)w. \tag{58}$$

From (57) and (58), we can calculate the relative welfare loss, which is given by

$$\frac{W^* - W}{W^*} = \frac{R+\mu-1}{R+\mu}L. \tag{59}$$

Quite intuitively, the welfare loss thus depends on the excess return on an asset, $R+\mu-1$. In addition, it is proportional to the amount of liquidity held in the economy. For our parameterization of the economy we have found that for $R+\mu = 1.1$ we have $L^* = 0.05$. This translates into a loss of around 0.5% using above equation.

In the internet appendix on endogenous leverage we also consider an economy with fixed costs of investment c per investor. Expected output in this economy is the same as before, except that there are now also fixed costs of Ic in total. We thus have for expected output:

$$W = (R + \mu)(1 - L)w + Lw - Ic. \tag{60}$$

First best output is unchanged as all investment can in principle be delegated to a single agent. Since a single agent's share in the economy is zero, his fixed costs do not affect welfare. From (60) we find that the welfare loss is now

$$\frac{W^* - W}{W^*} = \frac{R + \mu - 1}{R + \mu} L + \frac{c}{(R + \mu)w} I. \tag{61}$$

This is equal to the previous welfare loss (equation 59) plus the term $\frac{c}{(R+\mu)w} I$ that arises due to the fixed costs. For our parameterization of the economy ($R + \mu = 1.1$, $c = 0.014$ and $L^* = 0.05$) we obtain now a higher welfare loss of 1.7%.

Figure 5: Assumption 1 and the Lognormal and the Exponential Distribution

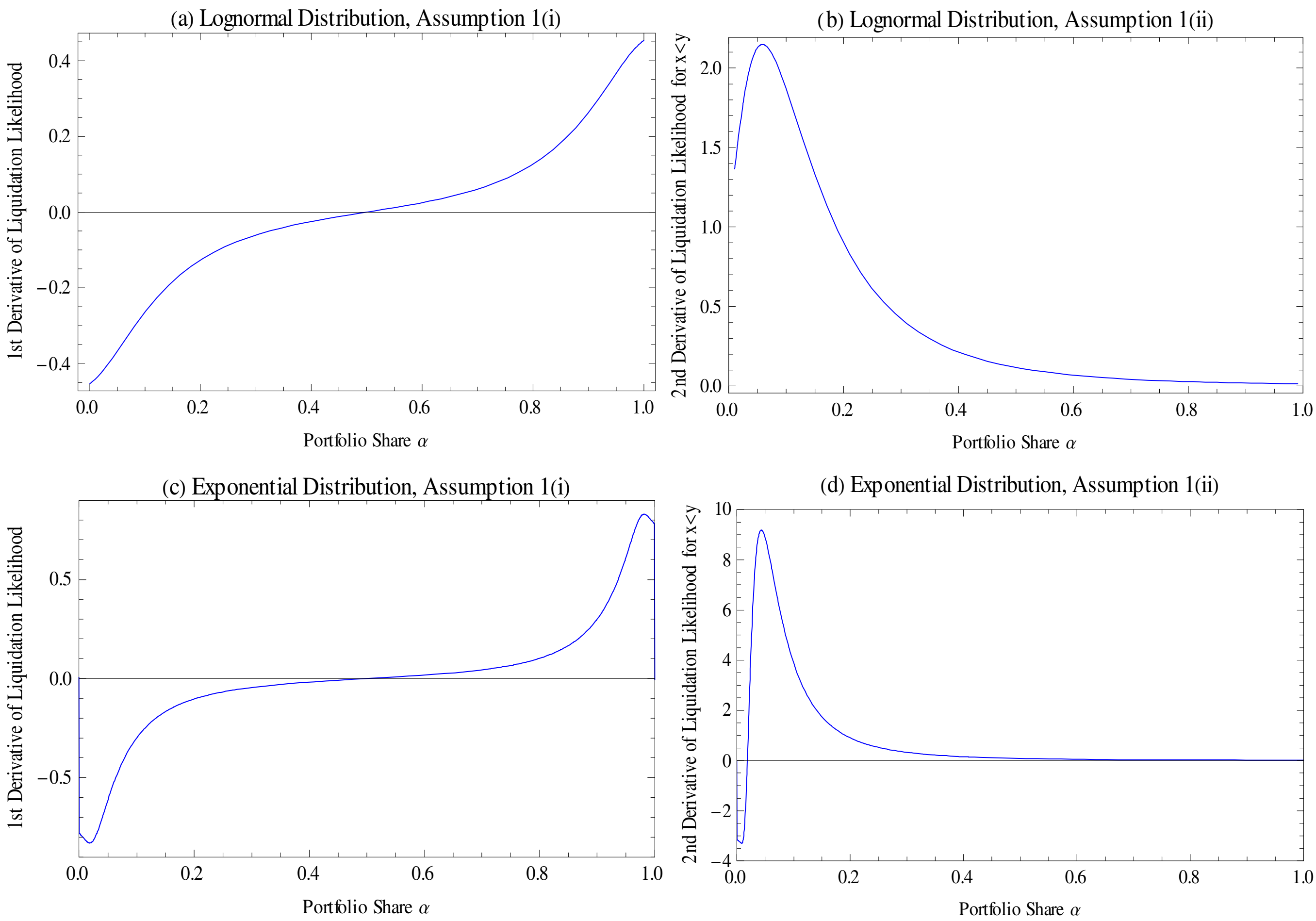

Figure 6: Endogenous Leverage